Simultaneous Search and Adverse Selection*

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Abstract

We study the effect of diminishing search frictions in markets with adverse selection by presenting a model in which agents with private information can simultaneously contact multiple trading partners. We highlight a new trade-off: facilitating contacts reduces coordination frictions but also the ability to screen agents' types. We find that, when agents can contact sufficiently many trading partners, fully separating equilibria obtain only if adverse selection is sufficiently severe. When this condition fails, equilibria feature partial pooling and multiple equilibria co-exist. We show that facilitating contacts can lead to a reduction in welfare. In the limit, as the number of contacts becomes large, some of the equilibria converge to the competitive outcomes of Akerlof (1970), including Pareto-dominated ones; other pooling equilibria continue to feature frictional trade in the limit, where entry is inefficiently high. Our findings provide a basis to assess the effects of recent technological innovations that have made meetings easier.

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1 Introduction

In this paper, we study an environment with two key ingredients: adverse selection and search frictions. Real-life markets that feature these ingredients are abundant and include labor markets, OTC markets, as well as insurance markets. In recent years, many of these markets have seen technological innovations giving rise to online platforms that made it easier for market participants to meet, thus lowering search frictions.[1] A natural question is how such innovations affect the strategies of traders and the resulting prices at which transactions occur and hence the properties of allocations obtained in those markets. An understanding of the welfare effects of lowering meeting barriers is important, also to guide possible regulatory interventions regarding the organization of trades in markets.

Our paper aims to provide a theoretical framework that allows us to investigate the question of how facilitating contacts affects market outcomes in the presence of adverse selection. The main innovation is to model reductions in search frictions by letting agents contact multiple potential trading partners *simultaneously* into an otherwise standard framework of directed search. We demonstrate that this, in the presence of adverse selection, gives rise to a new trade-off: facilitating contacts between market participants not only means search frictions are smaller but also affects the possibility of using the liquidity properties of different markets to screen traders with private information. We show that the latter effect has significant implications for the properties of market outcomes. In contrast to the case where agents can only contact one trading partner, equilibria in our setting may exhibit partial pooling where agents of different types trade at the same price. A striking result we obtain is that some of these equilibria feature inefficient entry, even in the limit when agents can contact infinitely many other market participants. Hence, frictional trade may persist in the limit where the exogenous search friction vanishes. Moreover, we demonstrate that—whether or not equilibria feature partial pooling—facilitating contacts can lead to a reduction in welfare.

Our analysis is cast in an environment as in Akerlof (1970), where sellers own an indivisible object and are privately informed about its quality, which can be either low or high. For expositional purposes, we adopt a labor market terminology throughout the paper: buyers

[1] As discussed by Fermanian et al. (2016) and Riggs et al. (2020), recent technological innovations (e.g., electronification) and regulatory changes (e.g., the Dodd-Frank Act) had a very significant impact on the way many securities are traded in financial markets. These innovations, together with measures aiming to increase transparency in trades, generated a substantial increase in contacts among market participants in OTC markets, where corporate bonds and derivatives like swaps are mostly traded. In the new platforms that emerged, customers can contact multiple dealers at the same time, both to have the quotes set by various dealers streamed to them (RFS) and to send a contemporaneous request for quote (RFQ) to a selected subset of dealers for a specific transaction.

are firms and sellers are workers who have private information about their productivity and can accept at most one job.[2] Our framework, however, is also applicable to other contexts; for example, we might reinterpret sellers as asset owners with private information about the asset's future value or as producers with private information about their products' quality.

The market works as follows. First, firms choose which wage to post, workers then send applications to $N \geq 1$ firms, and, finally, firms make offers. A worker's strategy thus specifies an *application portfolio*, trading off higher wages against the lower associated probabilities of getting a job offer. The matching between firms and workers is complicated by the fact that workers may receive multiple offers and can choose which one to accept. A firm's offer may thus be rejected. We assume that, when this happens, the firm can keep making new offers until an applicant accepts or the firm exhausts its applicant pool, as in Kircher (2009).

We now describe our results in more detail. The most novel and striking properties of equilibria are obtained when adverse selection is mild, meaning that the high types' outside option is smaller than the net productivity of low types (i.e., their productivity net of firms' entry cost). In this case, when workers can send sufficiently many applications, multiple equilibria exist, all of which feature low and high types sending a subset of their applications to the same firms. Hence, in equilibrium, there is at least one submarket where the two types of workers pool their applications, so full separation cannot be sustained. This result is in clear contrast with the properties of models à la Guerrieri et al. (2010), where meetings are bilateral and the unique equilibrium is always separating.

More specifically, we show that an equilibrium exists where low and high types send a subset of their applications to a *single* pooling market and characterize its properties. For the low-type workers, the wage in the pooling market is the highest to which they apply, while for the high types, it is the lowest. Hence, low types applying in the pooling market are hoping for a 'lucky punch,' whereas high types view jobs offered in this market as a fallback option in case their preferred applications fail to generate offers. With multiple applications, the high-type workers' opportunity cost of sending an application to a low wage is relatively small, and the same is true for low-type workers applying to a high wage.

When, instead, adverse selection is severe, there is always an equilibrium featuring com-

[2]While application data is scarce, the available evidence indicates that the number of applications sent by workers has increased in recent decades (see e.g. Martinelli and Menzio, 2020), likely facilitated by the increased use of online job search since the beginning of this century, as documented by e.g. Faberman and Kudlyak (2016). Further, as discussed by Wolthoff (2018), various pieces of evidence highlight the importance of simultaneous search. First, data from online job boards shows that workers tend to send multiple applications within even the shortest time span (a week or even a day). Second, surveys among employers indicate that the most common reason for a worker to reject a job offer is the simultaneous arrival of a more attractive offer. Informational asymmetries are also a natural feature of employment relationships, especially at the onset, even though they tend to be mitigated over time with on-the-job learning.

plete market segmentation, no matter how many applications workers can send. In such an equilibrium, workers of different types apply to different firms. To sustain sorting, the equilibrium probability of being hired in one of the high-type markets must be sufficiently low so as to ensure incentive compatibility for low types. The larger the number of applications low types have at their disposal, the tighter this constraint becomes. Multiple applications thus have another interesting implication: when adverse selection is severe, screening based on market liquidity may still occur but requires a much bigger distortion of the trading probability.

We then show that, in contrast to the case of symmetric information, the welfare implications of increasing the number of applications under adverse selection are ambiguous. The reason is that increasing the workers' application capacity not only relaxes search frictions but also affects the set of allocations that are incentive-compatible. While low-productivity workers always gain from an increase in capacity to apply, high-productivity workers may gain or lose. More specifically, focusing on the case where entry costs are small, we show that starting from the single-application benchmark, high-productivity workers lose from an additional application whenever adverse selection is severe or the fraction of low-productivity workers in the market is sufficiently large. Surprisingly, in the latter case, the loss occurs even though the additional application leads to partial pooling in equilibrium. For high-productivity workers, moving from a separating equilibrium when $N = 1$ to a partial pooling equilibrium when $N = 2$ reduces the average wage at which they are hired. When the average productivity in the pooling market is low (i.e., when the fraction of low-type workers is large), this more than offsets the benefit of the associated increase in their trading probability. These findings imply that it can be beneficial for a planner to restrict innovations that facilitate meetings among market participants.

Finally, we examine the properties of search equilibria in the limit, as the number of applications workers can send becomes large and exogenous search frictions vanish. We connect these limit results to the equilibrium outcomes in a perfectly competitive market à la Akerlof (1970). In our framework, a separating equilibrium exists for any number of applications workers can send if and only if the corresponding Akerlof economy has an equilibrium where only low-productivity workers are hired. We demonstrate that, as search frictions vanish, the probability of high-productivity workers being hired in a separating equilibrium converges to zero, making the Akerlof equilibrium the limit point of the separating search equilibria as $N \to \infty$. The other potential Akerlof equilibrium in a two-type economy is a pooling equilibrium, where both types of workers are hired at a wage equal to their average productivity. Considering the equilibrium with partial pooling under mild adverse selection, we find that, as the number of applications increases, the probability that workers are hired in the pooling market converges to one. There are, however, too many firms entering the market, so the firms' hiring probability is bounded away from one in the limit. Since, in equilibrium, firms must be compensated for their entry costs, workers bear the cost of excessive entry in the form of a wage below their average net productivity. As a result, equilibrium trading remains frictional, even in the limit as the number of applications workers can send becomes arbitrarily large. Nevertheless, we can construct alternative sequences of equilibria—featuring two pooling markets instead of one—that do converge to Akerlof's pooling equilibrium. Our findings thus show that, in our search-theoretic setting, convergence to the set of equilibria from Walrasian markets à la Akerlof (1970) is *possible but not necessary*, with multiplicity persisting in the limit as search frictions vanish.

The approach pursued in this paper differs from the one followed in most of the search literature, which restricts attention to bilateral meetings, either in a static environment or in (the steady state of) a dynamic environment. In these settings, a reduction in search frictions has been modeled as an increase in matching efficiency, captured either by the probability or the rate at which trading partners meet. Reducing search frictions in this way, however, has generally no qualitative impact on the nature of equilibrium outcomes. Indeed, for the case of directed search with adverse selection, Guerrieri et al. (2010) and Chang (2018) found that the key properties of equilibrium outcomes remain unchanged as the matching technology becomes efficient or sequential meetings become arbitrarily frequent: the limit equilibrium features perfectly segmented markets in which the lowest type trades with probability one, while higher types trade with a smaller but still positive probability. Our analysis shows that modeling declining search frictions as an increase in the number of simultaneous meetings affects the properties of equilibria in important ways and allows to get the competitive market outcome as a limit equilibrium allocation when search frictions vanish.

In a similar vein, Diamond (1971) demonstrated for the case of sequential random search à la McCall (1970) that when prices are endogenized, we obtain the surprising prediction that the equilibrium always features monopoly pricing and does not converge to the competitive outcome, even as search frictions become arbitrarily small. Butters (1977), Varian (1980) and Burdett and Judd (1983) showed that allowing buyers to meet multiple sellers simultaneously offers a resolution to the Diamond paradox. Specifically, they found that competitive pricing is obtained in the limit as the probability of meeting multiple trading partners converges to one. Part of our contribution is to provide analogous insights for the case of directed search with adverse selection.[3]

[3]We thank a referee for this valuable insight.

Related literature. Our paper contributes to various strands of literature. The two most related strands, as already discussed, concern models of simultaneous search and the work on adverse selection in directed search environments. The first strand includes Chade and Smith (2006), Albrecht et al. (2006), Galenianos and Kircher (2009), Wolthoff (2018) and Albrecht et al. (2020). Our model builds in particular on Kircher (2009), with respect to which we innovate by allowing for searcher heterogeneity and introducing asymmetric information. The second strand includes Gale (1996), Inderst and Müller (2002), Guerrieri et al. (2010) and Chang (2018), relative to which we innovate by allowing for simultaneous search.

Some recent papers have explored the interaction of search frictions and screening in random search models à la Burdett and Judd (1983). Garrett et al. (2019) study this issue in a canonical price discrimination problem with private values, as in Mussa and Rosen (1978). Lester et al. (2019) consider the case of adverse selection. In their setting, privately informed sellers randomly meet either one or two buyers. Buyers then optimize over screening menus, taking into account that sellers may receive better terms from another buyer. The probability with which a given seller meets two buyers captures the competitiveness of the market but can also be interpreted as an inverse measure of search frictions. Like us, Lester et al. (2019) are interested in the interaction between adverse selection and the level of search frictions.[4] Our work differs from theirs not only in how we model the decentralized market but also in terms of the predictions the two models deliver. When search frictions disappear in Lester et al. (2019), the equilibrium allocation converges to that of Rothschild and Stiglitz (1976), which is always separating. In contrast, lowering search frictions in our setting—via an increase in N—is what makes pooling possible when the lemons condition fails or the fraction of high-type workers is sufficiently large. Conversely, when search frictions are high, the equilibrium allocation in our setting is always separating, whereas in Lester et al. (2019) pooling becomes possible. The effect of lowering search frictions on market segmentation in the two settings thus goes in the opposite direction, leading to new and interesting welfare implications.

The consequences of multiple applications for the properties of equilibrium outcomes with search frictions have interesting analogies to those of non-exclusivity in contracting without such frictions. The latter also limits, though in different ways, the ability of firms to screen workers. Our environment features exclusivity in contracting, as each worker can accept only one offer, but not in applications. When firms compete with non-exclusive contract offers, Attar et al. (2011) find that pooling obtains in equilibrium under the same condition as in

[4] There are a number of other papers studying frictional markets with adverse selection that share some aspects of our framework but ultimately have a different focus; see, in particular, Kurlat (2016), Lauermann and Wolinsky (2016), Kim and Pease (2017), and Kaya and Kim (2018).

Akerlof (1970). Otherwise, high-quality sellers are excluded from trade and the equilibrium is unique. Our analysis differs in various respects. Unlike in Attar et al. (2011), firms in our model face a capacity constraint in their hiring, which limits their market power. As a result, the forces breaking separating equilibria and sustaining pooling trades in the two setups are rather different. Furthermore, multiplicity of equilibria and inefficient entry are distinctive phenomena of our analysis. Finally, our setup also allows us to investigate the properties of equilibrium outcomes where firms can only submit a finite number N of applications, so non-exclusivity at the application stage is only partial, gaining interesting insights on the effects of varying N.

The paper is organized as follows. Section 2 presents the framework. Some preliminary properties are derived in Section 3. Section 4 characterizes search equilibria—first for the case of severe adverse selection, then for the case of mild adverse selection—and discusses their welfare properties. Section 5 analyzes the equilibrium properties in the limit as search frictions vanish. Section 6 discusses policy implications and Section 7 concludes. Proofs and additional results are relegated to the (Online) Appendix.

2 Environment

Agents. We consider a static labor market populated by a continuum of size one of workers and a large continuum of firms. Both types of agents are risk-neutral. Workers supply and firms demand one unit of indivisible labor. All firms are identical, but workers differ in their productivity, defining their type, which is private information. In particular, a fraction σ of workers have low productivity, while the remaining ones are of high productivity. We will index types by $i \in \{L, H\}$.

Market interaction. The market interaction between workers and firms proceeds in multiple subsequent stages. In the first stage, firms decide whether to become active or not. Active firms incur an entry cost $k > 0$ and subsequently post and commit to a wage p that they will pay if they hire. The support of the distribution of posted wages is denoted by $\mathcal{F}$.

After observing all posted wages, in the second stage, each worker sends $N \in \{1, 2, \ldots\}$ job applications to firms.[5] As standard in the directed search literature (see e.g. Shimer, 2005) and motivated by the idea that coordination among a continuum of agents in decentralized markets is unrealistic, we restrict workers to symmetric and anonymous strategies, which creates the search frictions we study. That is, for each application, a worker selects a wage and then applies at random to one of the firms posting such wage. A worker's ap-

[5] We will generally focus on $N \geq 2$, but include $N = 1$ for completeness and comparison with the literature.

plication portfolio is thus a list of N wages. As we will show, whenever a worker has the opportunity to send an additional application, that application will be sent to a (weakly) higher wage than the previous ones. It is then convenient to order the applications sent in a weakly increasing order, so a portfolio is described by $(p_1, \ldots, p_N) \in \mathcal{F}^N$, with $p_1 \leq \ldots \leq p_N$. Although the worker sends all N applications simultaneously, it will often be useful to refer to p_n, i.e., the n-th lowest application, as the worker's n-th application.

After all the applications are sent, matches are formed. Following Kircher (2009), we assume that the matching of workers and firms on the network induced by the workers' applications is stable, i.e., no firm remains unmatched when one of its applicants is hired at a lower wage. One way to implement a stable outcome is via a sequential procedure in the spirit of deferred acceptance (Gale and Shapley, 1962). First, each firm with applicants randomly selects one of them and makes a job offer. Every worker then keeps the best job offer he received under consideration (without loss of generality, in the event of ties, we take this to be the offer with the highest index) and rejects all worse job offers. Next, firms whose job offers are rejected select a different applicant (as long as they still have one) and make a new job offer. After this, the process repeats until there are no more rejections or no firm can make any additional offers. At that point, workers accept the job offer under consideration.[6]

Finally, after matches are formed, production takes place and payoffs are realized. A match between a firm and a worker of type $i = L, H$ results in an output v_i, where $v_H \geq v_L$. The firm's payoff from the match is the difference between this output and the wage p it pays. In contrast, the worker's payoff from the match is the difference between this wage and his outside option (or disutility from effort) c_i. This also depends on the worker's type, with $c_H > c_L$, motivated by the idea that high-productivity workers have better opportunities in outside markets or self-employment. Unmatched workers and inactive firms receive a zero payoff. We assume the productivity of each type of worker, net of the firm's entry cost, exceeds the worker's outside option: $v_i - k > c_i$, $i = H, L$. Hence, there are positive gains from trade with each worker.

Queues. Consider a *(sub)market* $p \in \mathcal{F}$, defined as the collection of all the firms posting this wage and of all the applications they receive. From the firms' perspective, each application has two unobservable but payoff-relevant characteristics: i) its position $n \in \{1, \ldots, N\}$ in the sender's application portfolio, which affects the firms' matching probability, and ii) the type $i \in \{L, H\}$ of its sender, which affects the firms' payoff conditional on a match.

We define the *queue length* $\lambda_{n,i}(p) \in \mathbb{R}_+$ as the endogenous ratio of the number of ap-

[6]The same outcome can also be motivated as the result of a process in which the market clears from the top, i.e., firms posting the highest wages make job offers first, followed by firms posting the next highest wages, etc.

plications with characteristics (n, i) to the number of firms in submarket p. As common in the literature, the number of applications with characteristics (n, i) at a firm posting a wage p follows a Poisson distribution with a mean equal to this queue length, independently of the number of applications with other characteristics.[7] Some of these applications, however, will end up not being relevant for the firm as the worker will turn down a job offer due to receiving better offers from other firms. Denote by $\xi_{n,i}(p) \in [0, 1]$ the endogenous probability that an applicant with characteristics (n, i) accepts a job offer at p. The number of *effective* applicants with characteristics (n, i) at a firm posting wage p then follows a Poisson distribution with mean (or *effective queue length*) $\mu_{n,i}(p) = \xi_{n,i}(p)\lambda_{n,i}(p) \in \mathbb{R}_+$.

Payoffs. The probability that a firm posting a wage p hires a worker is equal to the probability the firm has at least one effective applicant, $\eta\left(\mu(p)\right) \equiv 1 - e^{-\mu(p)}$, where $\mu(p)$ is the total effective queue length in that market, obtained by aggregating the queues across all applicants' characteristics

$$\mu(p) \equiv \sum_n \sum_i \mu_{n,i}(p). \tag{1}$$

The probability that the hire is an L-type worker is equal to the effective fraction of L-type workers in the market:

$$\gamma(p) \equiv \sum_n \mu_{n,L}(p)/\mu(p) \tag{2}$$

The expected profit $\pi(p)$ of a firm offering wage p is then given by the probability of hiring a worker, times the difference between the worker's expected productivity and the wage, less the entry cost:

$$\pi(p) = \eta\left(\mu(p)\right)\left(\gamma(p)v_L + (1 - \gamma(p))v_H - p\right) - k. \tag{3}$$

Active firms choose a posted wage p so as to maximize their profit $\pi(p)$. Free entry implies that in equilibrium these profits are zero.

The expected payoff of a worker with application portfolio $(p_1, \ldots, p_N)$ depends on the total effective queue length at these wages. The worker obtains a payoff $p_n - c_i$ if two conditions are satisfied. First, the application to p_n results in a job offer, which happens with probability $\psi\left(\mu(p_n)\right) \equiv \eta\left(\mu(p_n)\right)/\mu\left(p_n\right)$. Second, none of the applications to higher wages $p_{n+1}, \ldots, p_N$ result in a job offer, which is the case with probability $\prod_{j=n+1}^{N}\left(1 - \psi\left(\mu\left(p_j\right)\right)\right)$. The expected payoff $u_{N,i}$ of a type i worker from the optimal choice of his N applications is then

$$u_{N,i} = \max_{(p_1,\ldots,p_N)\in\mathcal{F}^N} \sum_{n=1}^{N} \prod_{j=n+1}^{N} \left(1 - \psi\left(\mu(p_j)\right)\right)\psi\left(\mu_n(p)\right)\left(p_n - c_i\right).$$

[7]See Lester et al. (2015) and Cai et al. (2017) for a discussion of this property, which they call *invariance*.

As in Kircher (2009), this payoff can be rewritten in a recursive way, where

$$u_{n,i} = \max_{p \in \mathcal{F}} \psi\left(\mu(p)\right)(p - c_i) + \left(1 - \psi\left(\mu(p)\right)\right) u_{n-1,i} \tag{4}$$

is the payoff of the first n applications, for all $n \in \{1, \ldots, N\}$, and $u_{0,i} = 0$. Intuitively, the worker's n-th application leads to a wage offer p with probability $\psi\left(\mu(p)\right)$; with complementary probability, the worker does not receive such an offer, but still has the chance that one of his applications to lower wages is successful, yielding a conditional payoff equal to $u_{n-1,i}$. Since $u_{n-1,i}$ is the expected payoff from sending $n-1$ applications to wages below p_n and trading at those wages occurs with probability less than 1, it follows from the above equation that $u_{n,i}$ is strictly increasing in n. Going forward, we will refer to $c_i + u_{n-1,i}$ as the worker's *effective outside option* when sending his n-th application, and to $u_{n,i}$ as his *market utility* from sending n applications.

Beliefs. To decide whether to post a particular wage p, a firm needs to form beliefs about the applicant pool $(\mu(p), \gamma(p))$ that it will attract. Similarly, a worker considering applying to wage p chooses on the basis of beliefs over $\mu(p)$. If the wage is part of the equilibrium choices of firms, these beliefs are determined by the consistency conditions with firms' and workers' strategies, as described above. In particular, worker optimization implies that the effective queue length $\mu(p)$ must satisfy

$$u_{n,i} \geq \psi\left(\mu\left(p\right)\right)\left(p - c_i - u_{n-1,i}\right) + u_{n-1,i}, \tag{5}$$

with weak inequality for all (n, i) and with equality for at least one (n, i) if $\mu(p) > 0$.

If, instead, the wage is not part of the equilibrium, we follow the standard assumption in the directed search literature (see Wright et al., 2021) that these beliefs are pinned down by the *market utility condition*, which aims to capture the consequences of deviations in our continuum economy in the spirit of subgame perfection. In particular, the market utility condition extends the optimization condition (5) to all p that are not part of an equilibrium. That is, a firm posting $p \notin \mathcal{F}$ expects an effective queue length $\mu(p)$ implying the smallest job offer probability that is needed to induce one of the workers' types to redirect one of their applications to p, indeed in the spirit of subgame perfection. As in Guerrieri et al. (2010), this also pins down beliefs about the market composition: at this wage, the firm expects to attract applicants of a certain type only if (5) holds with equality for that type, in our case for some n. That is, for any $p \notin \mathcal{F}$, $\gamma(p)$ satisfies

$$\begin{cases} \gamma(p)\,\mu(p) = 0 & \text{if (5) holds with strict inequality for } i = L \text{ and all } n \\ (1-\gamma(p))\,\mu(p) = 0 & \text{if (5) holds with strict inequality for } i = H \text{ and all } n. \end{cases} \tag{6}$$

Equilibrium. We then define an equilibrium as follows.[8]

Definition 1. *An equilibrium is a set of wages $\mathcal{F}$ posted by firms, effective queue lengths and compositions $(\mu(p), \gamma(p))$ for all p, and market utilities $u_{n,i}$ for all n and i, such that the following conditions are satisfied.*

1. *Worker Optimization: a worker of type i sends his n-th application to wage $p \in \mathcal{F}$ only if (5) holds as equality;*
2. *Firm Optimization: $\pi(p) = 0$ for any $p \in \mathcal{F}$, and $\pi(p) \leq 0$ for any $p \notin \mathcal{F}$;*
3. *Consistency: for any $p \in \mathcal{F}$, $\mu(p)$ and $\gamma(p)$ are consistent with workers' and firms' strategies;*
4. *Out-of-Equilibrium Beliefs: for any $p \notin \mathcal{F}$, $\gamma(p)$ satisfies (6) and $\mu(p)$ satisfies (5) with weak inequality for any (n, i), and with equality for at least one (n, i) if $\mu(p) > 0$.*

In what follows, we shall refer to a submarket where only L-types (H-types) apply as an L-type market (H-type market).

3 Preliminaries

3.1 Indifference and Isoprofit Curves

Most of our analysis of workers' and firms' choices and hence of equilibria can be presented graphically by considering workers' indifference curves and firms' isoprofit curves. To facilitate this approach, we introduce these curves here and discuss some useful properties.

Isoprofit curves. As equation (3) shows, firms' profits depend not only on the price p and the effective queue length μ, but also on the queue composition γ. Hence, we need to specify the value of γ to be able to pin down a firm's isoprofit curve as the set of all combinations of μ and p satisfying the free entry condition. The two extremes in which the firm respectively

[8] To keep the notation simple in the main text, we state the consistency condition somewhat informally here. We provide the full details in Online Appendix C.

attracts only low types ($\gamma = 1$) or only high types ($\gamma = 0$) will prove to be particularly useful for our analysis. The isoprofit curves in those two cases are defined as

$$\Pi_i \equiv \left\{ (\mu, p) \in \mathbb{R}^2 : \eta(\mu)(v_i - p) = k \right\}, \tag{7}$$

with $i \in \{L, H\}$.

Indifference curves. The indifference curve $I_{n,i}$ of a worker of type i sending his n-th application consists of all combinations of μ and p that solve (5) with equality.

$$I_{n,i} \equiv \left\{ (\mu, p) \in \mathbb{R}^2 : \psi(\mu)(p - c_i) + (1 - \psi(\mu))\, u_{n-1,i} = u_{n,i} \right\}. \tag{8}$$

Differentiating (8) we see the slope of a worker's indifference curve equals

$$\frac{d\mu}{dp} = -\frac{\psi(\mu)}{\psi'(\mu)} \frac{1}{p - c_i - u_{n-1,i}} > 0.$$

It is useful to note that the slope depends on the type of the worker (only) through his effective outside option $c_i + u_{n-1,i}$. When the effective outside option varies with i, the two types have different marginal rates of substitution between wage and matching probability for their n-th application, which creates scope for screening. For the first application, this is always true since $u_{0,L} = u_{0,H} = 0$ and $c_H > c_L$. For subsequent applications $n = 2, 3, \ldots$, however, the effective outside option is endogenous. It is immediate to see from the above expression that a worker's indifference curve becomes steeper as the index n of the application increases. Intuitively, as the effective outside option of a worker increases, he is willing to tolerate a larger increase in the effective queue length to obtain a higher wage. It is also clear that, for the same index of the application, the high type has steeper indifference curves, i.e. $c_L + u_{n-1,L} < c_H + u_{n-1,H}$ for all n. What is less obvious, however, is how $c_L + u_{n-1,L}$ compares to $c_H + u_{m-1,H}$ for $n > m$. This question will be at the center of our analysis in the following section.

3.2 Observable Types

We start by describing the equilibrium allocation that arises if worker types are observable to firms and, hence, incentive constraints are absent, as obtained by Kircher (2009).

Equilibrium allocation. Due to free entry, the equilibrium allocation in this case can be determined for each worker type i in isolation. It is entirely pinned down by the free entry condition and the first-order condition of the firms' choice problem, taking into account beliefs as determined by market utility. Graphically, these beliefs are represented by the upper envelope of workers' indifference curves $I_{n,i}$, $n \in \{1,\ldots,N\}$. The effective queue lengths and wages for the N applications of each worker are then determined by the tangency points between the firms' isoprofit curve Π_i and this upper envelope, as illustrated in Figure 1. As shown by Kircher (2009), letting $p^*_{n,i}$ denote the wage to which a worker of type i sends his n-th application, one can combine these conditions to recursively characterize the equilibrium effective queue length $\mu^*_{n,i} \equiv \mu(p^*_{n,i})$ and the associated market utility $u^*_{n,i}$ for each application n and each type $i = L, H$. The procedure is as follows: set $u^*_{0,i} = 0$ and let $\{\mu^*_{n,i}, u^*_{n,i}\}_{i=1}^{N}$ be such that

$$k = \left(\eta\left(\mu^*_{n,i}\right) - \mu^*_{n,i}\eta'\left(\mu^*_{n,i}\right)\right)(v_i - c_i - u^*_{n-1,i}), \tag{9}$$

$$u^*_{n,i} = u^*_{n-1,i} + \eta'\left(\mu^*_{n,i}\right)(v_i - c_i - u^*_{n-1,i}). \tag{10}$$

Since the indifference curves become steeper as the index n of the application increases, the tangency point for this application moves up the firms' isoprofit curve to a higher wage and effective queue length.

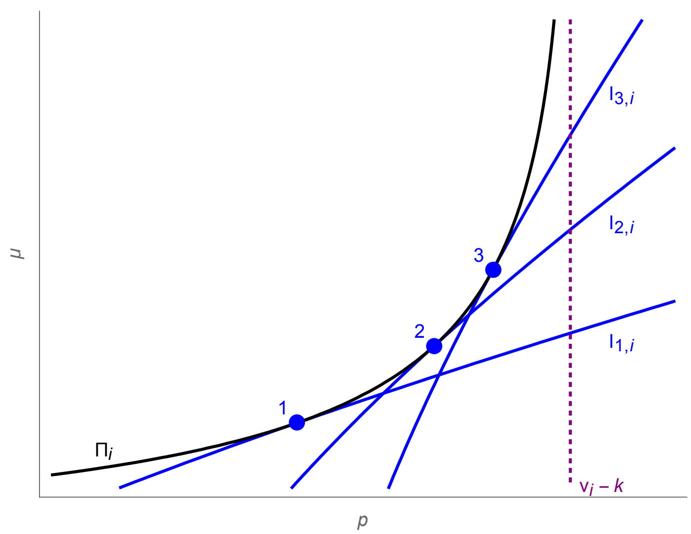

Figure 1: Equilibrium wages and effective queue lengths when the type is observable and workers send three applications.

As the number of applications N goes to infinity, we know from Kircher (2009) that the equilibrium allocation tends to the Walrasian outcome in which all firms active in the market hire a worker with probability one and every worker finds a job. To see this, notice first that the difference $u_{N,i} - u_{N-1,i}$ converges to zero as $N \to \infty$, since $u_{N,i}$ is strictly increasing in N and bounded above by the gains from trade, $v_i - c_i - k$. This property implies that the effective queue length $\mu^*_{N,i}$ tends to ∞, as can be seen from (10).[9] Since $\lim_{N\to\infty} \mu^*_{N,i} = \infty$,

[9] Since $u_{N,i} - u_{N-1,i} \to 0$, (10) implies that $\eta'(\mu(p)) = e^{-\mu(p)} \to 0$ and hence $\mu(p) \to \infty$.

every firm hires a worker, so the free-entry condition (9) requires that a worker's expected utility from his portfolio of applications, $u^*_{N,i}$, tends to $v_i - c_i - k$ as $N \to \infty$. Because no firm would offer a wage greater than $v_i - k$, this implies that, in the limit, a worker is hired at a wage $v_i - k$ with probability one. It further means that the measure of firms posting wages that are bounded away from $v_i - k$ (for which the effective queue length is finite) tends to 0. In other words, all entering firms hire with a probability of one in the limit. The impact of the search friction thus disappears in the limit where each worker can submit infinitely many applications, and the equilibrium allocation converges to the Walrasian outcome.

4 Equilibria with Adverse Selection

When types are unobservable, the allocation described in the previous section will often not be sustainable in equilibrium. The reason is that, due to the interdependence of values, L-type workers may find it profitable to send some applications to the submarkets designed for H-type workers. This point is illustrated in Figure 2, where we display the equilibrium allocation when both types are observable. Graphically, there are two relevant isoprofit curves for firms, one for hiring H-type workers and one for hiring L-type workers. The H-isoprofit curve is shifted to the right with respect to the L-isoprofit curve because, for each effective queue length μ, a firm is willing to pay a higher wage p for a worker of high productivity. In Figure 2, incentive compatibility is violated for L-type workers when they send $N \geq 2$ applications. They can gain, for instance, by sending their second application to the market where H-type workers send their first.

In what follows, we will study the implications of incentive constraints for the properties of the equilibrium allocation in our setting. How tight incentive constraints are will depend on the number of applications and the severity of adverse selection, i.e., the extent to which productivity and the workers' outside option depend on the workers' types. To account for this, we will split the equilibrium analysis into two parts.

4.1 Severe Adverse Selection

We first consider the case where adverse selection is severe, in the sense that the outside option of high-productivity workers is weakly greater than the net productivity of low types, i.e., $c_H \geq v_L - k$, or the *lemons condition* holds.[10]

Our first result shows that, as long as the lemons condition is satisfied, the fact that workers can send multiple applications does not fundamentally change the qualitative properties of search equilibria compared to the case where they can only send a single application.

[10] The lemons condition, as e.g. in Daley and Green (2012), is usually stated as $c_H > v_L$, for the case of no entry costs.

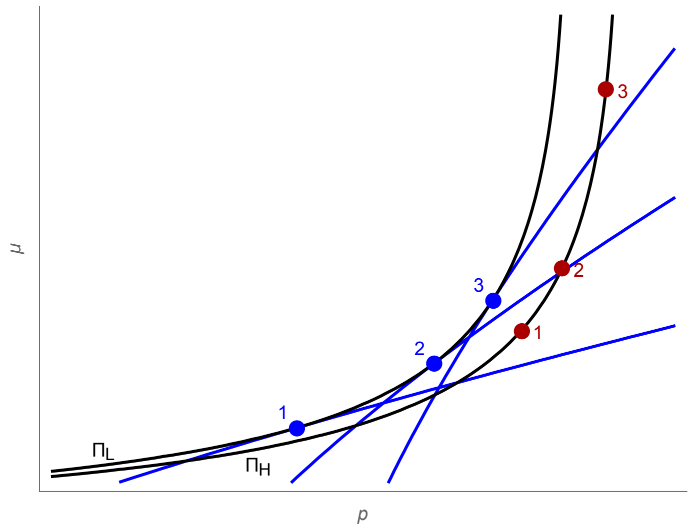

Figure 2: Equilibrium wages and effective queue lengths for the low type (blue) and the high type (red) when types are observable.

To recall, when workers apply to only one firm, we know from Guerrieri et al. (2010) and others that the unique equilibrium features complete market segmentation: L-type workers apply to a different market, with a lower price and a lower queue length, than the one to which high types apply. In our setting, the lemons condition guarantees that such sorting is sustained in equilibrium regardless of the number of applications workers can send.

Proposition 1. *If $c_H \geq v_L - k$, for all $N \geq 1$ there exists a separating equilibrium, where H-type workers apply to strictly higher wages than L-type workers.*

The construction of a separating equilibrium is as follows. The markets in which L-types apply are the same as in the equilibrium allocation of the observable type case: for all $n = 1, ..., N$, $(\mu_{n,L}, p_{n,L}) = (\mu^*_{n,L}, p^*_{n,L})$. The presence of H-type workers thus has no effect on the L-type allocation. In contrast, the wages and effective queue lengths of H-type markets are generally affected by the incentive constraints of L-type workers. We construct these markets sequentially. If the unconstrained solution for the H-types' first application, $(\mu^*_{1,H}, p^*_{1,H})$, satisfies the L-type incentive constraints, then the effective queue lengths and wages in all H-type markets are determined as in the observable type case, i.e., for all $n = 1, ..., N$, $(\mu_{n,H}, p_{n,H}) = (\mu^*_{n,H}, p^*_{n,H})$. If, on the other hand, $(\mu^*_{1,H}, p^*_{1,H})$ violates the L-type incentive constraints, $(\mu_{1,H}, p_{1,H})$ is given by the smallest effective queue length and wage on the isoprofit curve Π_H such that incentive compatibility holds. In this case, to find the market where H-types send their second application, we consider the tangency point between Π_H and the H-types' second indifference curve, i.e., the one corresponding to the effective outside option $c_H + u_{1,H}$. If this point satisfies $\mu_{2,H} > \mu_{1,H}$, incentive compatibility does not bind for the second application and the same is true for the remaining applications. Otherwise, incentive compatibility also binds for the H-typs' second application, in which case they send their first and second applications to the same market (see Figure 3). We repeat this procedure for the next applications until we reach the last application $n = N$.

By construction, the candidate separating equilibrium satisfies the incentive constraints of L-types. Hence, it constitutes an equilibrium as long as H-type workers have no incentives to send some of their applications to L-type markets. Since $p^*_{n,L} < v_L - k \leq c_H$ for all $n \geq 1$, wages in the L-type markets remain strictly below the high types' outside option, which means that those incentive constraints are always satisfied.

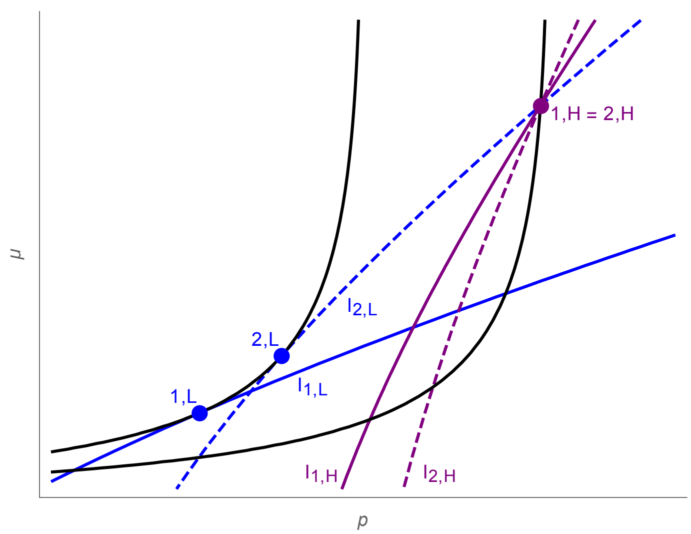

Figure 3: Equilibrium wages and effective queue lengths when workers send two applications, and incentive constraints bind for both applications of the high type.

4.2 Mild Adverse Selection

We now turn our attention to the most interesting case in our setup, where the lemons condition fails: $c_H < v_L - k$. Under this condition, we will show that allowing workers to send more than one application has two novel, important implications: *(i) equilibria with perfect market segmentation exist only if N is sufficiently small* and *(ii) pooling markets may be active in equilibrium*. Also, when (ii) occurs, there are typically multiple ways in which workers of different types can pool some of their applications, so the equilibrium is no longer unique.

We start by presenting the main result. The following proposition shows that there exists a threshold for the total number of applications N such that a fully separating allocation constitutes an equilibrium only when N is below that threshold, while a new type of equilibrium with a pooling market arises when N is above the threshold. The threshold is identified by the number of applications at which the L-type's payoff in the equilibrium of the benchmark case with observable types becomes larger than the H-type's outside option. Formally:

$$l \equiv \min\{n \in \mathbb{N} : u^*_{n,L} + c_L > c_H\}. \tag{11}$$

To state the result, we require that v_H is sufficiently distinct from v_L so that we are far enough from the private value case ($v_L = v_H$). This will imply, as we explain below, that in equilibrium H-types apply to weakly higher wages than L-types. In the proof of Proposition 2, we provide an explicit lower bound for v_H.

Proposition 2. *Assume $c_H < v_L - k$. There exists some $\bar{v}_H > v_L$ such that for all $v_H \geq \bar{v}_H$, the following holds.*

1. *A separating equilibrium where L- and H-types send their applications to distinct submarkets exists if and only if $N \leq l$.*
2. *If $N > l$, there exists an equilibrium with a single pooling market. In this equilibrium, L-types send their first l applications to separate submarkets and the remaining applications to the pooling market. In contrast, H-types send their last l applications to separate submarkets and their remaining applications to the pooling market.*

Separating equilibrium. We first look at the possible existence of a fully separating equilibrium. As we prove in the Appendix, any such equilibrium must follow the same construction as the one in Section 4.1.[11] This construction implies, as we already mentioned, that the incentive constraints of L-type workers are satisfied. However, since $\lim_{n\to\infty} p^*_{n,L} = v_L - k$ and $c_H < v_L - k$, some wages in the L-type markets are now acceptable to H-type workers when N is sufficiently large. Their incentive constraints must therefore also be examined. A separating equilibrium exists as long as H-type workers do not want to send any of their applications to one of the L-type markets.

A necessary and sufficient condition for this to be true is that the L-type workers' effective outside option associated with their last application, $c_L + u^*_{N-1,L}$, remains below the H-type workers' outside option, c_H, or equivalently, that $N \leq l$. When this holds, all indifference curves of L-type workers are flatter than those of H-type workers. This sorting condition is precisely the feature that allows firms to screen high-productivity workers on the basis of their greater propensity to accept longer queue lengths for higher wages. If, instead, N is strictly greater than l, then the sorting condition is reversed for the last application(s) of

[11] The property that H-type workers apply to strictly higher wages than L-type workers—even when the lemons condition fails—is again guaranteed by the condition $v_H \geq \bar{v}_H$.

L-type workers, and perfect market segmentation cannot be sustained in equilibrium.

Equilibrium with partial pooling. Proposition 2 shows that, when a separating equilibrium fails to exist ($N > l$), there is a partial pooling equilibrium where L-type workers and H-type workers send a subset of their applications to a single pooling market. Let us first recall why a pooling market cannot be sustained in equilibrium when $N = 1$. Consider a candidate equilibrium with a pooling market $(\bar{\mu}, \bar{p})$ attracting both L-type workers and H-type workers, as illustrated by the green point in Figure 4. Since firms attract both types, the isoprofit curve associated with zero profits lies between Π_L and Π_H, as illustrated by the green curve in the figure. Due to the higher outside option, the H-type indifference curve passing through $(\bar{\mu}, \bar{p})$ is steeper than that of the L-type. This difference in marginal rates of substitution implies that high types are willing to tolerate longer effective queue lengths than low types in any market with a wage higher than $\bar{p}$. In other words, an H-type worker has more to gain by applying to a wage above $\bar{p}$ than an L-type worker. If a firm deviates and increases the wage above $\bar{p}$, it thus expects to attract only H-type workers. A marginal increase in the wage and the associated queue length thus leads to a discrete improvement in the composition of the applicant pool and thus constitutes a profitable deviation, effectively a cream-skimming deviation.

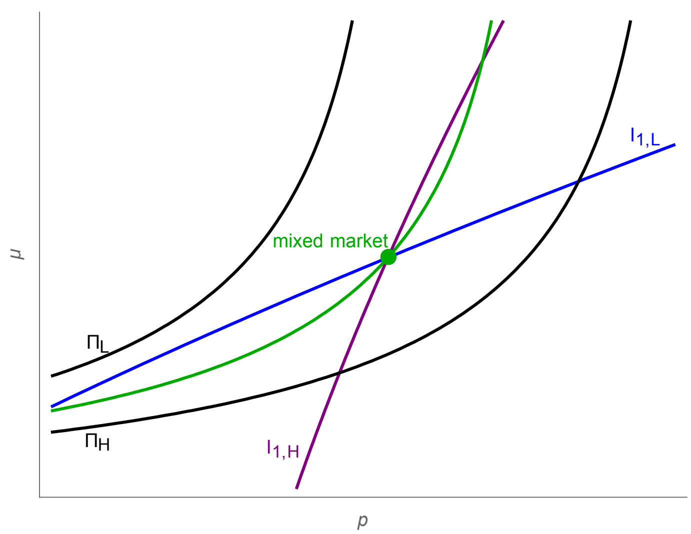

Figure 4: Cream-skimming deviation in the single-application case.

Proposition 2 shows that the cream-skimming argument breaks down when workers can send sufficiently many applications. Before discussing the construction in the general case, we graphically illustrate the equilibrium in Figure 5 for the special case where $N = 2$ and $l = 1$. There are three active markets: one with a low wage to which each low-type worker sends his first application $(1, L)$, one with a high wage to which each high-type worker sends his second application $(2, H)$, and one with an intermediate wage to which each low- (resp. high-) type worker sends his second (resp. first) application. We refer to the latter market as the *pooling market* since both types send applications there. Low types apply to the pooling market, hoping to receive an offer at the wage posted in that market but at the same time insuring themselves by also sending one application to a lower wage, where the chance of getting an offer is higher. In contrast, for high types, the pooling market represents the fallback option in case their application to a firm offering a higher wage fails. As in Figure 4, let $\bar{p}$ and $\bar{\mu}$, respectively, denote the wage and the effective queue length in the pooling market. Note, however, that now the (green) isoprofit curve is different from the one in Figure 4: the effective composition in the pooling market is not equal to the population average but worse than that because high types only agree to trade at the pooling wage $\bar{p}$ if they receive no offer in the high-wage market $(2, H)$.

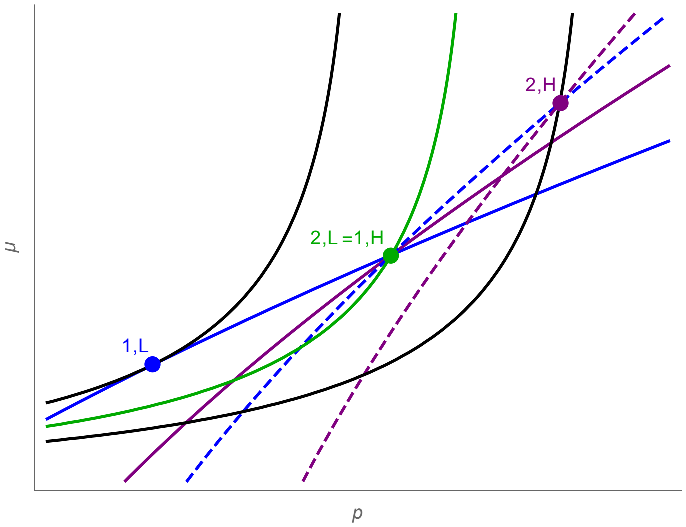

Figure 5: Equilibrium wages and effective queue lengths for $N = 2$.

To be able to claim that the described allocation constitutes an equilibrium, we need to verify that firms have no incentives to deviate by offering a different wage. In particular, we must rule out the profitability of cream-skimming deviations like the one we saw existed for the pooling allocation in Figure 4, when workers could only send one application. To assess the profitability of a deviation to a different wage, we must again determine which type of worker this wage is more likely to attract. In Figure 5, the L-types' indifference curve associated with their second application (the dashed blue curve) is steeper than the H-types' indifference curve associated with their first application (solid purple curve). This happens because in the case of $l = 1$, the L-types' effective outside option for their second application, $c_L + u^*_{1,L}$, is higher than the H-types' outside option for their first application, c_H. We thus have a *reversal of the sorting condition* relative to Figure 4: this implies that it is not the high types who have the most to gain from applying to wages slightly above $\bar{p}$ but rather the low types with their second application. Hence, a firm contemplating offering one of those wages expects to attract only L-type workers, rendering the deviation unprofitable. For wages below $\bar{p}$, it is again the low types who have the most to gain, this time by sending their first application. Hence, firms can only worsen the composition of the set of workers they attract by deviating to a wage slightly above or below $\bar{p}$, which means that no profitable cream-skimming deviation exists.[12]

The construction in Figure 5 can then be generalized as follows. Low and high types send, respectively, their first and their last l applications to separate markets, while all remaining applications are sent to a single pooling market with wage $\bar{p}$. The terms of trade in the L-type markets are the same as when their type is observable, while the terms of trade in the pooling market are such that L-types are indifferent between sending their l-th application to the pooling market or to the respective L-type market (as in fig:mixed). The wages and effective queue lengths in the H-type markets are determined by the L-types' N-th incentive constraint. They can be constructed via the sequential procedure explained in Section 4.1.

In the proof of Proposition 2, we show that deviating to a price slightly below or above $\bar{p}$ does not allow a firm to improve the composition of the applicant pool in market $\bar{p}$. This requires $u^*_{l-1,L} + c_L < c_H$ and $u_{N,L} + c_L \geq u_{N-l,H} + c_H$, so that for wages just below $\bar{p}$ the one who gains most is the L-type redirecting his l-th application and for wages just above $\bar{p}$ it is the L-type deviating from his N-th application. While the first inequality is satisfied by the definition of l, the second follows from $u^*_{l,L} + c_L \geq c_H$ and the fact that both types of workers send $N - l$ applications to the pooling market and, therefore, have the same chance of receiving an offer there. Hence, firms deviating to wages slightly below or above $\bar{p}$ expect to attract only low types. Finally, we need to verify that firms do not find it profitable to attract L-type workers at any off-path wages. This requires that the L-types' indifference curves in the candidate equilibrium do not intersect the isoprofit curve Π_L. Our assumption that v_H is sufficiently large implies that this property is satisfied.[13]

[12] The reversal of the sorting condition is somewhat reminiscent of the violation of the single crossing condition found by Chang (2018) in a competitive search model with two-dimensional private information. In that case, the outside option of traders is exogenous but subject to idiosyncratic (liquidity) shocks. In our setup, it is instead endogenous, so a given type of seller will have different preferences depending on which of his applications we are considering. The implications for the analysis of equilibria are then rather different.

[13] For a numerical example of the bound for v_H, consider the specification $v_L = 2, c_L = 0, c_H = 1, k = 0.1, N = 2$. It can be verified that $u^*_{1,L} + c_L \approx 1.40 > 1 = c_H$, so we have $l = 1$. Hence, the candidate equilibrium, as described in Proposition 2, is of the partial pooling type (for all $v_H > v_L$ and $\sigma \in (0, 1)$). For $\sigma = 0.5$, the bound for v_H above which the partial pooling equilibrium is guaranteed to exist is $\bar{v}_H \approx 2.33$. If the share of low-type workers is reduced to $\sigma = 0.3$, this bound decreases to $\bar{v}_H \approx 2.23$. Hence, the higher the share of high-productivity workers, the more permissive the condition on v_H becomes. When v_H lies below this bound, an equilibrium with a single pooling market may not exist, as one of the L-type indifference curves going through the pooling market may intersect the L-type isoprofit curve. In such a case, firms have

Remark 1. Besides the equilibrium we illustrated in Figure 5 and whose existence we established more generally in Proposition 2, other partial pooling equilibria may exist. For instance, if the fraction of high types is sufficiently large, one can construct equilibria with a single pooling market where low types send fewer than l applications to separate markets and a larger number of applications than high types to a pooling market. In these equilibria, the composition of applicants in the pooling market is strictly worse than in the equilibrium we constructed. However, a fully pooling equilibrium in which both types send all their applications to the same market never exists. In that case, the same cream-skimming deviation argument as in the one-application case applies.

4.3 Welfare Implications

When workers' types are publicly observable, the only effect of increasing workers' application capacity is to alleviate the search friction and thereby increase their trading probability. Hence, the equilibrium welfare of all workers unambiguously increases with N. In contrast, when workers are privately informed about their type, increasing N not only mitigates the search friction but also affects incentive-compatible trades. In particular, we saw it diminishes the screening role of market liquidity since workers can hedge against the possibility of not being hired in an illiquid market by sending some of their applications to more liquid markets. Hence, in markets with adverse selection, a trade-off arises: allowing workers to submit multiple applications, on the one hand, reduces the search friction but, on the other hand, restricts the possibility of screening workers. This has interesting welfare implications, as we discuss now.

The fact that screening workers becomes more difficult as N increases can manifest itself in two ways. First, in the separating equilibrium, a larger N leads to increasingly congested H-type markets, which may reduce the likelihood of high types being hired (despite their increased capacity to apply). Second, the difficulty of screening workers via queue lengths may destroy the possibility of separation altogether, making partial pooling a necessary feature of the equilibrium. To illustrate the welfare implications in both situations, we will focus on the simplest case, where the number of applications workers can send increases from $N = 1$ to $N = 2$. When $N = 1$ the equilibrium is always separating with two active markets, whereas for the case $N = 2$ we saw that two outcomes are possible: if $c_H < v_L - k$ and $l = 1$, there is an equilibrium in which workers send one of their applications to a pooling market; otherwise, the equilibrium remains separating.

a profitable deviation by posting a wage higher than the pooling wage to attract low types. A partial pooling equilibrium can still be found for values $v_H < \bar{v}_H$, but a different, more elaborate construction involving multiple pooling markets is needed.

Considering the latter case, it is clear that L-type workers always benefit from an increase in the capacity N. Since their equilibrium allocation is the same as in the observable type case, their expected payoff must increase. With regard to the welfare of high types, there are two countervailing effects. An increase in N, on the one hand, reduces the chance of any single application being successful since, due to the steeper slope of $I_{2,L}$ relative to $I_{1,L}$, the incentive constraints imposed on the trading probability of H-type workers are tighter. On the other hand, by sending two applications, high types can at least partly offset the risk that any given application fails. The first effect is not present—hence high types gain when N increases—if the incentive constraints of the low-type workers are slack so that the terms of trade in the H-type markets are not affected by these constraints. This is true, for instance, if the gains from trade with H-type workers, $v_H - c_H - k$, are sufficiently small.

In contrast, a sufficient condition for H-type workers to unambiguously lose when moving from $N = 1$ to $N = 2$ is a sufficiently small entry cost k. When entry costs are negligible, low types already extract almost all the gains from trade with their first application. The opportunity cost of sending their second application to one of the high-type markets is then very small, which means they are deterred from doing so only if the trading probability in such markets is close to zero. Hence, when workers send two applications, high-type workers are essentially driven out of the market. It follows that the ex-ante welfare of workers is lower for $N = 2$ than for $N = 1$ since low types gain very little from the additional application, while the payoff loss for high-type workers is non-negligible.

A similar picture arises when an increase from $N = 1$ to $N = 2$ implies a switch from the separating equilibrium to an equilibrium with partial pooling (that is, when $c_H < v_L - k$ and $l = 1$). Since L-type workers are better off when they can pool with H-type workers, their gain from an increase in the application capacity is even larger in this case. The welfare effect for H-type workers remains ambiguous and now depends on the share of L-type workers in the economy. To illustrate this, let us focus again on the limit case where the firms' entry cost vanishes. In this case, $c_H < v_L - k$ is sufficient to ensure that the sorting reversals occurs after the first application, i.e., $l = 1$.[14] Letting $U_H(N)$ denote the equilibrium utility of H-type workers when the total number of applications workers can send is N, we can show the following:

Proposition 3. *For any k sufficiently small, there exists some $\sigma_k \in (0, 1)$ such that $U_H(1) \leq U_H(2)$ if and only if $c_H < v_L - k$ and $\sigma < \sigma_k$.*

We already explained above why, for k sufficiently small, $U_H(1) \leq U_H(2)$ is violated when the lemons condition holds, so we focus our discussion here on the case $c_H < v_L - k$.

[14] As already mentioned, for k small, L-type workers extract almost all the gains from trade with their first application: since $\lim_{k\to 0} u^*_{L,1} + c_L = v_L > c_H$ and $c_H < v_L - k$, we indeed have $l = 1$.

In the limit as $k \to 0$, the separating equilibrium for $N = 1$ has low types trading with probability one at price v_L. Similarly, the wage in the H-type market approaches v_H as $k \to 0$, but, due to incentive compatibility, the workers' trading probability in that market is strictly smaller than one.[15] In the case of $N = 2$, the equilibrium features partial pooling. The terms of trade in the L-type market where low types send their first application remain unchanged, while the queue length in the H-type market where high types send their second application is higher than in the single-application case. Whether high types gain or lose from an increase in the application capacity depends then on whether the benefit of sending the additional application to the pooling market outweighs the cost of facing a higher queue length in the H-type market. For the gain to dominate, the wage in the pooling market has to be sufficiently high. This requires the average quality of workers in the pooling market to be sufficiently good and, hence, the fraction of low-productivity workers in the population to be sufficiently low.

5 Vanishing Search Frictions

As N grows, an increasing number of firms and workers become connected, leading to a reduction in the underlying search friction. In the limit, as $N \to \infty$, the exogenous search friction disappears altogether. When worker types are observable, this implies that the search equilibrium allocation converges to the Walrasian outcome in which all active market participants find a trading partner with probability one (Kircher, 2009), as we explained above. With adverse selection, the Walrasian outcome, where traders are price takers, have private information and search frictions are absent, was characterized by Akerlof (1970). Recall that with binary types (and entry costs) there are two potential equilibria when markets are as in Akerlof: a separating one where only low types trade at price $v_L - k$ and a pooling one where both types trade at price $\sigma v_L + (1-\sigma)v_H - k$. The separating equilibrium exists when the lemons condition holds, while the pooling equilibrium exists if $c_H \le \sigma v_L + (1-\sigma)v_H - k$. When $c_H \in [v_L - k, \sigma v_L + (1-\sigma)v_H - k]$, the Akerlof economy thus has two equilibria.

Severe adverse selection. Proposition 1 shows that when the lemons condition holds ($c_H \ge v_L - k$), a separating equilibrium exists for all N, exactly as in Akerlof (1970). In this equilibrium, the queue lengths in the H-type markets are distorted upwards so that L-types have no incentives to reallocate any of their applications to one of the H-type markets. As the number of applications grows, L-type incentive constraints become increasingly tight,

[15] The limit case $k \to 0$ is another way of modeling vanishing search frictions. As argued in the Introduction and seen here, for $N = 1$, the qualitative features of the search equilibrium are unaffected when k vanishes.

thereby pushing up the queue length in the markets where high-productivity workers search. Intuitively, when workers have a large number of applications at their disposal, the opportunity cost of diverting one of these applications to an H-type market becomes very small. Incentive compatibility then requires that, as $N \to \infty$, the queue length in the markets to which H-types apply tends to ∞. As the number of applications grows, H-type workers thus face increasingly congested markets. The following proposition shows that, in the limit, the increase in congestion outweighs the larger number of feasible applications, so that H-type workers are eventually driven out of the market. Hence, the allocation of the search equilibrium converges to that of the separating equilibrium in the competitive market à la Akerlof (1970).

Proposition 4. *Assume $c_H \geq v_L - k$. As $N \to \infty$, the probability that an H-type worker is hired in a separating equilibrium tends to zero. The market utilities for L- and H-type workers take the following limits:*

$$\lim_{N \to +\infty} u_{N,L} = v_L - c_L - k,$$
$$\lim_{N \to +\infty} u_{N,H} = 0.$$

To prove the result, we consider a candidate separating equilibrium where H-type workers are hired with a strictly positive probability and then construct a profitable deviation for low types. If low types follow the equilibrium strategy and send all their applications to the respective L-type markets, their probability of being hired tends to one and their wage to $v_L - k$. Suppose instead an L-type worker sends half of his applications to the first $N/2$ L-type markets and the remaining applications to the H-type market with the lowest effective queue length. Since N is arbitrarily large, the probability of being hired in one of the L-type markets is still arbitrarily close to one and the wage is arbitrarily close to $v_L - k$. We then show that sending half of the applications to the H-type market allows the L-type worker to be hired in that market with strictly positive probability. Since the wage in the H-type market is greater than c_H, which in turn is greater than $v_L - k$, this portfolio of applications generates a strictly higher expected wage and thus constitutes a profitable deviation.

Mild Adverse Selection. Turning our attention to the case where the lemons condition fails ($c_H < v_L - k$), we know from Proposition 2 that, for N sufficiently large, there is a search equilibrium where low- and high-productivity workers send some of their applications to a common submarket. Moreover, the characterization in Proposition 2 tells us that when the application capacity N increases, the number of applications that are sent to separate markets remains unchanged and equal to l, so all additional applications go to the single pooling market. As $N \to \infty$, the L-type indifference curves for the last applications become increasingly steep, which implies that the queue lengths in the H-type markets tend to ∞ (see Figure 6). As a result, the probability with which an H-type worker is hired outside the pooling market converges to zero, and the effective composition in the pooling market converges to the population average. Since the pooling market lies on the L-type indifference curve associated with the l-th application, the effective queue length in the pooling market remains finite in the limit, as illustrated in Figure 6.

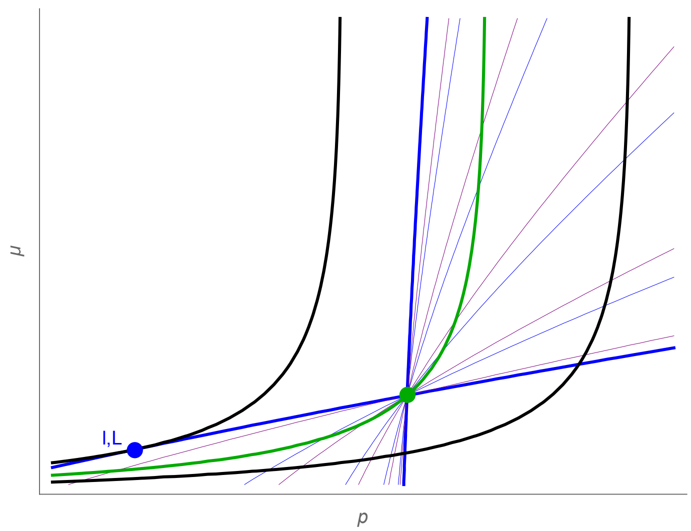

Figure 6: Wages and effective queue lengths in the equilibrium with one pooling market.

This last property has significant implications for the characteristics of the allocation obtained in the limit. Since the number of applications low and high types send to the pooling market tends to infinity as $N \to \infty$, a finite value of μ implies that the probability with which any type ends up receiving an offer in the pooling market converges to one, as in the pooling equilibrium of Akerlof (1970). In contrast, the probability that a firm hires a worker in the pooling market remains bounded away from one, and the wage remains bounded away from $\sigma v_L + (1-\sigma)v_H - k$. Hence, there is excessive entry in the limit.[16] This is an important result as it shows that, in the presence of adverse selection, the inefficiency of the search equilibrium may not vanish in the limit and the convergence to an equilibrium of the corresponding Walrasian market is not guaranteed.

[16] The result may suggest that a regulator would find it beneficial to tax entry in the pooling market. This policy, however, would leave a smaller surplus to workers, who would need to be compensated by a higher chance of being matched, thus exacerbating the excessive entry problem. Graphically, we see in Figure 6 that a tax on the pooling market would move northwest the green isoprofit curve, thus moving to the left its intersection point with the indifference curve $I_{l,L}$, associated with the L-type's l-th application. What the regulator should do is somewhat the opposite: tax entry in the L-type markets and subsidize entry in the pooling market. The tax in the L-type markets leads to higher queue lengths in those markets, pushing upwards the indifference curve $I_{l,L}$ and thereby increasing the queue length in the pooling market.

Proposition 5. *Assume $c_H < v_L - k$. Then, as $N \to \infty$, at the equilibrium characterized in Proposition 2, the workers' probability of being hired in the pooling market converges to one and their market utility satisfies*

$$\lim_{N\to\infty}(\sigma u_{N,L} + (1-\sigma)u_{N,H}) < \sigma(v_L - c_L) + (1-\sigma)(v_H - c_H) - k, \tag{12}$$

hence there is excessive entry in the limit.

Proposition 5 follows directly from the equilibrium characterization in Proposition 2 for the case $N > l$ and the arguments provided above. In Appendix B.1, we show that the result extends to all sequences of search equilibria with a single pooling market, as long as v_H is not too close to v_L.

Intuitively, efficiency would require that the queue length in the pooling market tends to ∞ and the price approaches $\sigma v_L + (1-\sigma)v_H - k$ as the number of applications per worker grows. To see that this is incompatible with equilibrium, consider a candidate sequence of equilibria with a single pooling market $(\bar{\mu}_N, \bar{p}_N)$ and $\bar{\mu}_N \to \infty$ as $N \to \infty$. H-type workers in the equilibria of this candidate sequence send some of their applications to the pooling market and the remaining applications to H-type markets with even higher wages and queue lengths. Consider then the H-type workers' first application and recall that the tradeoff workers solve when deciding where to send this application does not depend on where they send the remaining applications—the problem is as if they only had this one application. In the one-application problem, a wage associated with a very large queue length is extremely unattractive, so sending the first application to that market will be optimal for H-type workers only if all markets with lower queue lengths have wages below c_H. Suppose now a firm deviates and posts a wage $\tilde{p} \in (c_H, v_L - k)$. As $\bar{\mu}_N \to \infty$, the queue length H-type workers are willing to tolerate for them to divert their first application to wage $\tilde{p}$ will tend to ∞ as well. By the market utility condition pinning down out of equilibrium beliefs, a firm posting $\tilde{p}$ thus expects to hire with a probability close to one. In principle, it could be L-type workers, not H-type workers, who have the most to gain from changing their application portfolio and applying to the new wage $\tilde{p}$, so the deviating firm may expect to attract only low-productivity workers. But, since $\tilde{p} < v_L - k$, even in this case, the deviation yields a strictly positive profit for the firm, thus breaking the equilibrium.

Other Equilibria. When pooling markets can be sustained in equilibrium, multiple equilibria exist. In Remark 1 we already discussed the existence of multiple equilibria with a single pooling market, but multiplicity can also take other forms. This raises the question of whether there exist alternative sequences of search equilibria that do converge to the efficient pooling equilibrium of Akerlof (1970) when the lemons condition is violated, or, more generally, when c_H is below the average net productivity $\sigma v_L + (1-\sigma)v_H - k$. The following result shows that this is indeed possible. Hence, even if not *all* sequences of equilibria converge to a Walrasian equilibrium as in Akerlof (1970), any equilibrium of Akerlof (1970) can be obtained as the limit of *some* sequence of search equilibria as $N \to \infty$.

Proposition 6. *Assume $c_H < \sigma v_L + (1-\sigma)v_H - k$. There exists a threshold $\bar{v}_H$ such that for all $v_H \geq \bar{v}_H$ the following holds.*[17] *For each $\varepsilon > 0$ arbitrarily close to zero, we can find some N_ε such that for all $N > N_\varepsilon$, an equilibrium exists where*

$$\sigma u_{N,L} + (1-\sigma)u_{N,H} \geq \sigma(v_L - c_L) + (1-\sigma)(v_H - c_H) - k - \varepsilon. \tag{13}$$

When $c_H < v_L - k$ we showed in Proposition 5 that whenever the equilibrium features a single pooling market, efficiency fails in the limit because the effective queue length in the pooling market is finite, meaning that too many firms enter the market. In the proof of Proposition 6, in the Online Appendix, we show that, under the same parameter conditions, a sequence of equilibria with two pooling markets exists where, as $N \to \infty$, the probability that both workers and firms are matched converges to one. The key idea is the following. The first pooling market takes care of workers' incentives to hedge by sending some applications to firms posting lower wages. But now a second pooling market is active and the wage in this market increases with the total number of applications so that the queue length approaches infinity in the limit. Since almost all applications are sent to the second pooling market, firms' entry is efficient in the limit. The construction is illustrated in Figure 7.

Turning attention to the case $c_H \in [v_L - k, \sigma v_L + (1-\sigma)v_H - k)$, it is easy to see that incentive compatibility for high types is not an issue here. Since the lemons condition is satisfied and the wage in any L-type market is below $v_L - k$, H-type workers will never want to send any of their applications to such markets.[18] This implies that we can let the index of the first application that L-types send to the pooling market grow with N to get an arbitrarily high effective queue length in this market. In the limit, firms then hire with probability one at a wage equal to the average net productivity. Note further that by varying how fast the index of this first pooling application grows with N, one can support many additional equilibria in the limit, whose outcomes lie between the two equilibrium allocations found by Akerlof, featuring complete pooling and complete separation.

To sum up, for any equilibrium in Akerlof (1970), we can find a sequence of search equilibria that converges to the same allocation as $N \to \infty$. When $c_H < \sigma v_L + (1-\sigma)v_H - k$ holds,

[17] The bound for v_H needed in this result is the same as the one of Proposition 2; see Appendix A.2 for an explicit characterization.

[18] We used this property to establish the existence of a separating equilibrium when the lemons condition holds, in Proposition 1.

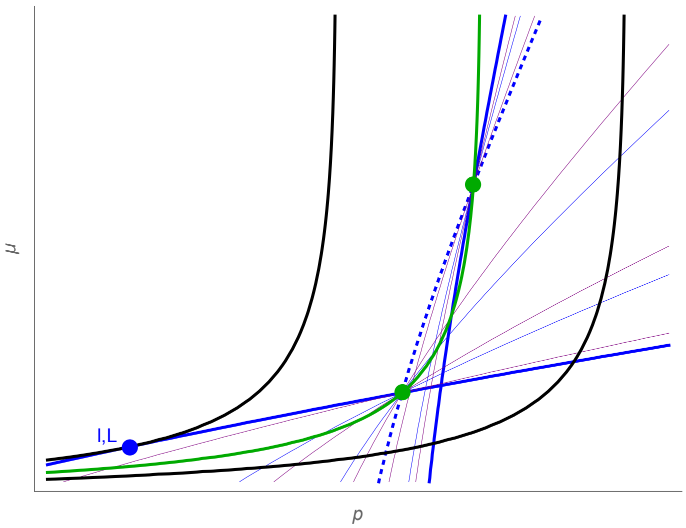

Figure 7: Wages and effective queue lengths in an equilibrium with two pooling markets.

multiple equilibria exist and the multiplicity persists in the limit as search frictions vanish. In particular, there are many sequences of equilibria that fail to converge to an equilibrium allocation of the corresponding Akerlof (1970) economy and feature frictional trade in the limit. This stands in contrast with the findings of Kircher (2009) for the observable type case, where search equilibria are always unique. More strikingly, it also stands in contrast with the results obtained when firms compete strategically in contract offers without search frictions. Both Attar et al. (2011) with general contracts under non-exclusivity and, e.g., Mas-Colell et al. (1995) with exclusive contracting when traded quantities are restricted to $\{0, 1\}$ find a unique equilibrium outcome, given by the Pareto-dominant equilibrium allocation in Akerlof (1970).

A key role behind this difference in equilibrium outcomes is played by the fact that in our environment, firms are interested in hiring at most one worker, i.e., firms compete for workers but face a capacity constraint. Without any capacity constraints, firms would find it profitable to deviate from the separating equilibrium by posting higher wages to attract all workers. In contrast, doing so with a limited hiring capacity would only attract the workers who are most keen to apply for higher wages, and these are the low types.[19] Our analysis, therefore, suggests that the decentralized property of markets and the fact that traders have limited market power have important consequences in environments with adverse selection.

Remark 2. Competitive search models with adverse selection and a single opportunity to contact a potential trading partner, such as Guerrieri et al. (2010), feature the stark property that the equilibrium outcome does not depend on the type distribution. Thus, the presence

[19] What ultimately matters is the presence of *some* capacity constraint, not that the capacity is one.

of low types severely distorts the equilibrium allocation for high types, even if the fraction of low types in the population is very small. The discontinuity in this allocation at the point where such fraction is zero is sometimes viewed as unappealing (see, for example, Lester et al., 2019). The same criticism applies to our model if we consider the separating equilibrium. However, as Proposition 6 shows, if σ is close to 0, other equilibria with partial pooling exist and the efficient outcome can be approached in the limit as $N \to \infty$. Hence, if we focus on the most efficient equilibrium, we can say that, compared to the single-application benchmark, the discontinuity becomes smaller when workers can send multiple applications and disappears when $N \to \infty$.

6 Policy Implications

A natural question is to what extent policy can improve equilibrium outcomes in our environment. The answer of course depends on the instruments available to the policymaker. Consider, for example, a policymaker who can only control the degree of search frictions in the market (i.e., set the level of N), e.g., by creating a trading platform that facilitates meetings. As already mentioned, a higher value of N is always welfare-improving under perfect information (Kircher, 2009). It readily follows from the discussion above that this is no longer true with adverse selection: depending on parameter values, $N = 2$ may lead to either higher or lower ex-ante welfare than $N = 1$. A similar picture emerges when we compare $N = 1$ with $N \to \infty$. Intuitively, although search frictions inhibit trade, they can serve a socially useful role in environments with adverse selection by facilitating screening. The policymaker has to balance these forces and will, therefore, not always prefer larger values of N. This result is consistent with the presence of application caps in various real-life markets. For example, the number of universities that high-school students in the UK can apply to was six until 2008. After an evaluation by the Department for Education and Skills (2006), this number was then reduced to five in order to improve the process. Similarly, workers can apply to at most 4 of the 15 different schemes in the United Kingdom's Civil Service Fast Stream per annual recruitment cycle.

Alternatively, imagine a policymaker that takes N as given but has the power to determine which submarkets are active, e.g., by imposing price controls, and to affect the tightness of these markets, e.g., by imposing taxes or subsidies on entry. By doing so, the policymaker operates effectively as a social planner who assigns workers and firms to different submarkets subject to search frictions, incentive constraints, and individual rationality constraints. For instance, the policymaker could implement perfect pooling by assigning all firms and workers to a single submarket. In many environments with adverse selection, pooling—or, more generally, some cross-subsidization between types—is socially beneficial as it allows to relax incentives. Davoodalhosseini (2019) establishes this formally for the environment where $N = 1$ considered by Guerrieri et al. (2010). When N is larger than 1, the planner's problem is more complex and new trade-offs emerge.

To illustrate this, we consider the case in which the outside options of the two types of workers are close to each other and workers send two applications. In this case, we obtain a partial-pooling equilibrium as in Figure 5: it features 3 markets, (μ_L, p_L), $(\bar{\mu}, \bar{p})$ and (μ_H, p_H), with low types applying to the first two and high types applying to the last two. The following proposition establishes that the planner can always increase ex-ante welfare with respect to the equilibrium by creating two separate submarkets designed in such a way that both types of workers choose to send one application to each market. Furthermore, this allocation strictly dominates full pooling, i.e., the creation of a single submarket in which all workers send all their applications. We show in the proof that no taxes or transfers are needed in order to achieve a higher level of ex-ante welfare, i.e., price controls suffice.

Proposition 7. *For c_H sufficiently close to c_L, a planner designing two markets can generate a higher level of ex-ante welfare than both the market equilibrium and a planner who designs a single market (thus implementing full pooling of types and applications).*

Hence, relative to the partial-pooling equilibrium, the planner does not improve welfare by implementing full pooling but rather by implementing a different kind of partial pooling. In particular, the planner chooses to pool the first application of both types in one market and the second application of both types in another.[20] Intuitively, there are two forces. On the one hand, as in the $N = 1$ case, there is a benefit in pooling the applications of different types of workers. On the other hand, there is a benefit in separating the different applications of each worker, because each application has a different effective outside option, as shown by Kircher (2009) when agents' types are observable.

7 Conclusion

We study a market in which firms post wages to attract applications from workers who have private information about their productivity. We demonstrate how increasing contacts in such a market not only decreases search frictions but also reduces firms' screening ability. The subtle interaction between these forces generates a rich set of outcomes. In particular, we find that—in contrast to a situation where each worker can send a single application—the existence of a fully separating equilibrium is only guaranteed if adverse selection is sufficiently

[20]Note that this would be the equilibrium outcome if c_L were actually equal to c_H. Since workers would have identical preferences in that case, firms would have no way of separating them. The equilibrium allocation would then be the same as in the observable type case, described in Section 3.2, with the average productivity $\sigma v_L + (1 - \sigma) v_H$ replacing the type-specific one v_i.

severe. When this condition fails, and workers can send sufficiently many applications, the equilibrium features pooling markets and multiple equilibria exist.

We analyze the properties of these equilibria as the number of applications grows large and, hence, search frictions vanish. While the allocation in the separating search equilibrium converges to the one with Walrasian markets à la Akerlof (1970), the same is not generally true for equilibria with pooling markets: some of them exhibit frictional trade and thus inefficiency in the limit due to excessive entry. Finally, we show that, with adverse selection, the welfare consequences of facilitating contacts among market participants are ambiguous. New policy implications, some of them surprising, can be drawn from our analysis.

In our model, we exogenously fixed the number of applications L- and H-type workers can send and assumed this number is the same for both types. However, the benefits of sending additional applications are generally different for the two types of workers. Hence, if workers could choose how many applications to send, facing a fixed cost per application, H-and L-type workers may make different choices. In Online Appendix D, we extend the analysis to this case and show that the total number of applications sent by L-type workers is, in fact, higher than that sent by H-type workers. As a consequence, high types send fewer applications to separate markets and, in many situations, do not trade in such markets, even away from the limit. Hence, accounting for workers' application incentives on the extensive margin reinforces the effect of adverse selection in our environment. Apart from this difference, we show that the main properties of equilibrium allocations remain valid when the number of applications sent by each type is endogenously determined.

An interesting avenue for future research would be to extend our model to a dynamic setting. We expect that our results remain unchanged if matches are long-lived, the unemployment pool is stationary and firms do not learn the types of their hires. Learning would add a new interesting dimension and could, for example, endogenously generate heterogeneity in outside options. We therefore expect that the trade-offs characterized in our paper will remain important. In a labor market context, an interesting question would also be to what extent the insights of our model could be generated through on-the-job search rather than simultaneous search, as these mechanisms are known to often have similar effects (see Burdett and Mortensen (1998) versus Burdett and Judd (1983) in random search, or Delacroix and Shi and Menzio and Shi (2010) versus Kircher (2009) in directed search).[21]

[21] We thank Guido Menzio for this insight.

Appendix A Proofs

In what follows, we will denote by

$$I_i(u_{n-1,i}, u_{n,i}) = \{(\mu, p) : \psi(\mu)(p - c_i - u_{n-1,i}) = u_{n,i} - u_{n-1,i}\}$$

type $i = L, H$'s indifference curve associated with utility levels $u_{n-1,i}, u_{n,i}$ and by

$$\Pi_\gamma = \{(\mu, p) : \eta(\mu)(\gamma v_L + (1-\gamma)v_H - p) = k\}$$

the firms' isoprofit curve when the fraction of L-types is γ.

A.1 Proof of Proposition 1

We first show that in any separating equilibrium, the terms of trade in an L-type market attracting the n-th application of low-type workers is described as a tangency point of the isoprofit curve Π_L and the indifference curve $I_L(u_{n-1,L}, u_n, L)$. By Condition 1 of Definition 1, we have $(\mu_{n,L}, p_{n,L}) \in I_L(u_{n-1,L}, u_{n,L})$. Assuming now that $(\mu_{n,L}, p_{n,L})$ was not a tangency point of $I_L(u_{n-1,L}, u_n, L)$ with Π_L, there would exist some $(\tilde{\mu}, \tilde{p}) \in I_L(u_{n-1,L}, u_n, L)$ such that

$$\eta(\mu(\tilde{p}))(\gamma(\tilde{p})v_L + (1-\gamma)(\tilde{p})v_H - p) \geq \eta(\tilde{\mu})(v_L - \tilde{p}) > k,$$

where the first inequality follows from $\gamma(\tilde{p}) \in [0, 1]$ and, by the market-utility condition,

$$\mu(\tilde{p}) = \max\{\mu' \geq 0 : (\mu', p) \in I_i(u_{n-1,i}, u_{n,i}) \text{ for some } i = L, H, n = 1, ..., N\}.$$

Hence, firms would have a profitable deviation, so all active L-type markets $(\mu_{n,L}, p_{n,L}), n = 1, .., N$ must be tangency points of the respective indifference curves with the isoprofit curve Π_L. Starting from the L-types' first application, where the outside option is endogenous, an iterative argument then implies $\mu_{n,L} = \mu^*_{n,L}, p_{n,L} = p^*_{n,L}$ and $u_{n,L} = u^*_{n,L}$ for all $n = 1, 2, \ldots, N$. The L-types' allocation in a separating equilibrium thus coincides with the one obtained for the observable type case.

Next, we consider the H-type markets in a candidate separating equilibrium. Given the lemons condition $c_H \geq v_L - k$, there is a unique intersection between the upper envelope of the low type's indifference curves $I_L(u^*_{n-1,L}, u^*_{n,L}), n = 1, ..., N$ and Π_H. This intersection is with $I_L(u^*_{N-1,L}, u^*_{N,L})$. Hence, the only incentive constraint potentially binding is the one associated with the L-type's N-th application. Let $(\underline{\mu}_H, \underline{p}_H)$ be the (unique) value of $(\mu_{n,H}, p_{n,H}) > (\mu^*_{N,L}, p^*_{N,L})$ satisfying $(\mu_{n,H}, p_{n,H}) \in I_L(u^*_{N-1,L}, u_{N,L})$ and $(\mu_{n,H}, p_{n,H}) \in \Pi_H$.

Suppose first $\mu^*_{n,H} \geq \underline{\mu}_H$ for all $n \geq 1$. In this case incentive constraints are not binding. We set for all n, $\mu_{n,H} = \mu^*_{n,H}$ and $u_{n,H} = u^*_{n,H}$. Notice that the associated wages satisfy $p^*_{1,L} < p^*_{2,L} < ... < p^*_{N,L} < p^*_{1,H} < p^*_{2,H} < ... < p^*_{N,H}$. For each p, we set

$$\mu(p) = \max\{\mu : \psi(\mu)(p - c_i - u_{n-1,i}) \leq u^*_{n,i} - u^*_{n-1,i} \text{ for some } i \in \{L, H\},\, n \leq N\}$$

and $\gamma(p) = 0$ for all p such that the previous max is attained at $i = H$ and $\gamma(p) = 0$ otherwise. It can be easily verified that this specification of the functions μ and γ satisfies the market utility condition.

If $\mu^*_{1,H} < \underline{\mu}_H$, we follow a recursive procedure to find the effective queue lengths and market utilities in the H-type markets. We start by setting $\mu_{1,H} = \underline{\mu}_H$ and $u_{1,H} = \psi(\mu_H)(\underline{p}_H - c_H)$. Given $u_{1,H}$, we calculate the unconstrained solution of $\mu_{2,H}$. Setting $n = 2$, the solution is determined by

$$(1 - e^{-\mu_{n,H}} - \mu_{n,H} e^{-\mu_{n,H}})(v_H - c_H - u_{n-1,H}) = k \tag{A.1}$$

If the value of $\mu_{2,H}$ solving this condition is weakly greater than $\underline{\mu}_H$, it is the effective queue length in market $(2, H)$. The associated market utility is

$$u_{n,H} = e^{-\mu_{n,H}}(v_H - c_H) + (1 - e^{-\mu_{n,H}})u_{n-1,H} \tag{A.2}$$

The queue lengths and market utilities of the remaining markets ($N > 2$) are then determined by the same set of conditions.

If instead $\mu_{2,H}$ solving (A.1) for $n = 2$ is strictly smaller than $\underline{\mu}_H$, we set $\mu_{2,H} = \underline{\mu}_H$. The market utility $u_{2,H}$ is then determined by (A.2). We repeat the procedure for all $n > 2$. Having fixed market utilities in this way, the functions μ, γ can be specified as follows. For all $p < \underline{p}_H$ we set $\gamma(p) = 1$ and for all $p \geq \underline{p}_H$ we set $\gamma(p) = 0$. For wages $p < \underline{p}_H$, the queue length $\mu(p)$ is then determined as the upper envelope of the indifference curves $I_L(u^*_{n-1,L}, u^*_{n,L}), n = 1, ..., N$; for wages $p \geq \underline{p}_H$, it is determined as the upper envelope of the indifference curves $I_H(u_{n-1,H}, u_{n,H}), n = 1, ..., N$ with $\{u_{n,H}\}^N_{n=1}$ specified by the recursive procedure.

No profitable deviations. The incentives of L-types are satisfied by construction. The equilibrium thus exists if and only if 1) H-types cannot profit from sending a subset of their applications to one of the L-type markets, and 2) firms cannot profit from posting an alternative wage.

Regarding incentive compatibility of H-type workers, notice that wages in the L-type markets are bounded above by $v_L - k$. The assumption $c_H \geq v_L - k$ then implies that the wages in the L-type markets are strictly below the H-type's outside option, hence sending applications to such markets cannot be profitable for H-types, independent of how many application workers can send.

As to requirement 2, recall that we set $\gamma(p) = 1$ for all off-path wages $p < \underline{p}_H$ and $\gamma(p) = 0$ for all $p \geq \underline{p}_H$ so as to satisfy the market utility condition (6). Since the L-type indifference curves, $I_L(u^*_{n-1,L}, u^*_{n,L})$, $n = 1, ..., N$, are tangent to Π_L, and the H-type indifference curves are either tangent (if L-type incentives are slack) or cut Π_H from below (if L-type incentives bind), this implies

$$\eta(\mu(p))(\gamma(p)v_L + (1 - \gamma(p))v_H - p) \leq k,$$

as can be easily verified. Hence, in the candidate separating equilibrium, no market participant has incentives to deviate. □

A.2 Proof of Proposition 2

Separating equilibrium. The candidate separating equilibrium follows the same construction as in Appendix A.1: L-type markets are as in the observable-type case, H-type markets are constructed recursively so as to satisfy the L-types' incentive constraint with respect to their N-th application. Given that v_H is sufficiently large (see below for the construction of an explicit bound), there is again a unique intersection between the upper envelope of the low type's indifference curves $I_L(u^*_{n-1,L}, u^*_{n,L}), n = 1, ..., N$ and Π_H, namely with $I_L(u^*_{N-1,L}, u^*_{N,L})$. The separating equilibrium constructed in Appendix A.1 is thus the only candidate for such equilibrium.

No profitable deviations. By the same argument as in Appendix A.1, L-types and firms have no profitable deviation. In contrast to the case of severe adverse selection, the H-types' incentive constraints, however, may now be violated. Letting $\psi_{n,i} \equiv \psi(\mu_{n,i})$ denote the probability of receiving an offer in market i, n, incentive compatibility generally requires that for any $(n, i) \neq (n', i')$ with $u_{n,i} + c_i \leq u_{n',i'} + c_{i'}$, we have $\psi_{n,i} \geq \psi_{n',i'}$, which follows from standard arguments. Incentive compatibility for the L-type requires that $\mu_{1,H}$ is weakly greater than $\underline{\mu}_{1,H}$, which is strictly greater than $\mu^*_{n,L}$ for all $n \leq N$. This is then compatible with the incentive constraints of H-types if and only if $u^*_{N-1,L} + c_L \leq c_H$, or, equivalently, if and only if $N \leq l$. Hence, given $c_H < v_L - k$ and v_H sufficiently large, a separating equilibrium exists if and only if $N \leq l$.

Partial Pooling Equilibrium. We begin the proof by constructing a candidate equilibrium where L-types send the last m applications to the pooling market, while H-types send the first m' applications to that market and, for now, we allow m to differ from m'. The first $N-m$ applications of the L-type are sent to separate markets, which are the same as in equilibrium with observable types (or the separating equilibrium). Hence, the effective queue lengths and market utilities in these markets are $\mu_{n,L} = \mu^*_{n,L}$ and $u_{n,L} = u^*_{n,L}$, for all $n \leq N-m$.

We will first determine the effective queue lengths and wages in the pooling market and the H-type markets, taking as given the number of applications the two types send to the pooling market, m and m', and the composition in that market, given by the effective fraction $\bar{\gamma}$ of L-type workers. Let $\bar{\mu}$ and $\bar{p}$ be, respectively, the effective queue length and the wage in the pooling market. We set their values to be such that the L-type is indifferent between sending the $N-m$-th application to market $(N-m, L)$ and sending it to the pooling market.

The terms of trade in the pooling market $(\bar{\mu}, \bar{p})$ must then satisfy

$$(\bar{\mu}, \bar{p}) \in \left(\Pi_{\bar{\gamma}} \cap I_L(u^*_{N-m-1,L}, u^*_{N-m,L})\right). \tag{A.3}$$

It is easy to verify that this condition has a unique solution on the domain $(\bar{\mu}, \bar{p}) > (\mu^*_{N-m,L}, p^*_{N-m,L})$. Let us denote such value with $\bar{\mu}(\bar{\gamma}), \bar{p}(\bar{\gamma})$ and set $\bar{\mu} = \bar{\mu}(\bar{\gamma}), \bar{p} = \bar{p}(\bar{\gamma})$.

To find the utility gains L- and H-types attain by trading in the pooling market, it is useful to define the probability of receiving an offer in a market with effective queue length μ when sending $n \geq 1$ applications to that market:

$$\beta(n, \mu) := 1 - (1 - \psi(\mu))^n \tag{A.4}$$

The market utility of H-type workers associated with their first m' applications is then $u_{n,H}(\bar{\gamma}) = \beta(n; \bar{\mu}(\bar{\gamma}))(\bar{p}(\bar{\gamma}) - c_H), n = 1, ..., m'$, while the market utility of L-type workers associated with their last m applications is $u_{N-m+n,L}(\bar{\gamma}) = \beta(n; \bar{\mu}(\bar{\gamma}))(\bar{p}(\bar{\gamma}) - c_L) + (1 - \beta(n, \bar{\mu}(\bar{\gamma})))u^*_{N-m,L}, n = 1, ..., m$.

To determine the separating markets to which H-types send their $(m+1)$-th and subsequent applications, let $(\mu_H(\bar{\gamma}), p_H(\bar{\gamma}))$ be the unique solution of

$$(\mu_H, p_H) \in (\Pi_H \cap I_L(u_{N-1,L}(\bar{\gamma}), u_{N,L}(\bar{\gamma}))$$

satisfying $(\mu_H, p_H) > (\bar{\mu}, \bar{p})$. Note that $p_H(\bar{\gamma})$ is the lowest wage to which only H-types are willing to apply. We then need to compare the utility they attain by sending applications to $p_H(\bar{\gamma})$ and to higher wages, at which incentive constraints no longer bind. When H-types send $n \geq 1$ applications to market $p_H(\bar{\gamma})$, they attain a utility level

$$u_{m'+n,H} = \beta(n; \mu_H(\bar{\gamma}))(p_H(\bar{\gamma}) - c_H) + (1 - \beta(n, \mu_H(\bar{\gamma})))u_{m',H}. \tag{A.5}$$

If the solution for μ of

$$(1 - e^{-\mu} - \mu e^{-\mu})(v_H - c_H - u_{m'+n-1,H}) = k \tag{A.6}$$

is greater than $\mu_H(\bar{\gamma})$, this means that the unconstrained solution for the $(m' + n)$-th application (starting from reservation utility $u_{m'+n-1,H}$) is feasible and hence preferred to market $p_H(\bar{\gamma})$. Let $\bar{n}$ be the lowest value of n for which this happens, that is, at which the L-type incentive constraint no longer binds. In equilibrium H-types will then send $\bar{n} - 1 \geq 0$ applications to wage $p_H(\bar{\gamma})$. For all $n \geq \bar{n}$, we set $\mu_{m'+n,H}(\bar{\gamma})$ equal to the unconstrained solution, solving (A.6) for a level of the market utility $u_{m'+n,H}$ determined by (10), starting from the value $u_{m'+\bar{n}-1,H}$ pinned down by (A.5). Set then $\mu_{m'+n,H}(\bar{\gamma})$ equal to $\mu_H(\bar{\gamma})$ for $n = 1, .., \bar{n} - 1$ and to the unconstrained solution, solving (A.6), for $n = \bar{n}, ..N - m'$.

Using these values we can derive the value of the probability with which an H-type worker is not hired in one of the H-type markets as a function of the effective composition $\bar{\gamma}$ in the pooling market:

$$\tau_H(\bar{\gamma}; m') = \prod_{n=1}^{N-m'} (1 - \psi(\mu_{m'+n,H}(\bar{\gamma}))). \tag{A.7}$$

For any given $m, m' \geq 1$, the effective composition $\bar{\gamma}$ in the pooling market is determined by:

$$\bar{\gamma} = \frac{\sigma m}{\sigma m + \tau_H(\bar{\gamma}; m')(1 - \sigma)m'} \tag{A.8}$$

To see that (A.8) has a solution, for any m, m', notice that both the left-hand side and the right-hand-side are continuous in $\bar{\gamma}$ on $(0, 1)$.[22] Since $\tau_H(\bar{\gamma})$ belongs to $(0, 1)$, the value of the right-hand side belongs to the interval $(\frac{\sigma m}{\sigma m + (1-\sigma)m'}, 1)$. As $\bar{\gamma} \to 0$ the left-hand side is then strictly smaller than the right-hand side, which is always greater than $\frac{\sigma m}{\sigma m + (1-\sigma)m'}$. In contrast, as $\bar{\gamma} \to 1$, the left-hand side is strictly greater than the right-hand side, since for any given N, m, m', $\lim_{\bar{\gamma} \to 1} \tau_H(\bar{\gamma}; m') > 0$. Hence, a solution of (A.8) always exists, constituting a candidate equilibrium for any $m, m' \geq 1$.

No profitable deviations. We want to show that for $N > l$, the candidate equilibrium with the specification $m = m' = N - l$ exists. Incentive constraints of workers are satisfied by

[22] It is immediate to verify that the map $\mu_H(\bar{\gamma})$, defined above, is continuous in $\bar{\gamma}$, while for $n \geq \bar{n}$ the map $\mu_{m'+n,H}(\bar{\gamma})$ is in fact independent of $\bar{\gamma}$.

construction. Hence, what we need to show is that firms cannot gain by posting an off-path wage. The following lemma pins down the off-path beliefs regarding the composition of workers in the candidate equilibrium. The proof is in the Online Appendix.

Lemma 8. *Consider the candidate equilibrium constructed in Appendix A.2 with $m = m' = N - l$. For all $p \in [0, \bar{p})$ and $p \in (\bar{p}, p_H)$, we have $\gamma(p) = 1$.*

According to Lemma 8, firms believe to attract the L-type when posting a wage below $\bar{p}$. Single crossing and L-type's indifference between sending the l-th application to $p^*_{l,L}$ and $\bar{p}$ imply that for any $p < \bar{p}$, the pair $(p, \mu(p))$ belongs to the upper envelope of the indifference curves of the L-type's first l applications. This property and $\gamma(p) = 1$ imply that there is no $p < \bar{p}$ such that $\eta(\mu(p))(v_L - p) > k$.

For wages p belonging to $(\bar{p}, p_H)$, the queue length $\mu(p)$ is such that

$$(\mu(p), p) \in I_L(u_{N-1,L}, u_{N,L}). \tag{A.9}$$

A sufficient condition for a deviation to such wages not to be profitable is that prices above $\bar{p}$ are higher than the net productivity of L-type workers, i.e., $\bar{p} \geq v_L - k$. This condition is satisfied whenever the productivity of H-type workers is sufficiently high, so that the composition in the mixed market is sufficiently favorable (again, for the construction of an explicit bound for v_H, see below). In this case, the isoprofit curve Π_L and the indifference curve $I_L(u_{N-1,L}, u_{N,L})$ have no intersection, so all pairs $(p, \mu(p)) \in I_L(u_{N-1,L}, u_{N,L})$ yield a negative profit for firms.

Finally, standard arguments imply that firms do not want to deviate to wages $p > p_H$ (where $\gamma(p) = 0$), as such a deviation would constitute a move away from the unconstrained solution of the problem of attracting H -types, with reservation utility $u_{N-l,H}$. □

Bound for v_H. We now construct an explicit threshold $\bar{v}_H$ such that for v_H above this threshold the statement of Proposition 2 holds. To this end, we will construct two thresholds, the first assuring that firms cannot attract high types to wages below $p^*_{l,L}$ without violating any L-type incentive constraints, the second one assuring that firms cannot profit from attracting low types at wages above $\bar{p}$ in the candidate equilibrium with partial pooling.

For the first threshold, let us introduce the L-type's lower contour set in the observable type allocation, i.e. all the pairs (μ, p) that the L-type worker does not prefer to $(\mu^*_{n,L}, p^*_{n,L})$ for all $n \in \mathbb{N}$:

$$U_L \equiv \left\{(\mu, p) \geq (\mu^*_{1,L}, p^*_{1,L}) : \forall n \in \mathbb{N},\ \psi(\mu)\left(p - c_L - u^*_{n-1,L}\right) \leq u^*_{n,L} - u^*_{n-1,L}\right\}.$$

If the difference between v_H and v_L is sufficiently close to or equal to zero, the set U_L has a non-empty intersection with the H-type isoprofit curve, Π_H. As v_H grows, the intersection of the two sets shrinks. Let v_H^0 be then the largest value of v_H such that $U_L \cap \Pi_H \neq \emptyset$. Not that, for $v_H > v_H^0$, the only incentive-compatible pairs (μ, p) yielding zero profits with the H-type are the points in Π_H lying above the intersection with $I_{N,L}$.

For the second threshold, notice that the probability with which H-types do not receive an offer in the H-type markets in the candidate equilibrium with partial pooling is bounded below by $\tau_H(\sigma, 1)$, as specified in (A.7), the probability associated to a specification where both types send one application to the pooling market and the composition in that market is given by the population average. For any $N > l$, the actual number of applications going to the pooling market is greater than one, while the composition is worse than the population average ($\bar{\gamma} > \sigma$). Both changes tighten the incentive constraint and thus imply a higher probability of H-types not receiving an offer in one of the H-type markets. For all $N > l$, we thus have $\tau(\bar{\gamma}, N - l) > \tau(\sigma, 1)$, with $\bar{\gamma}$ determined as the fixed point of (A.8). Since $\tau(\sigma, 1)$ is a lower bound for the probability that H-types do not receive an offer in the H-type markets, the composition parameter associated to $\tau(\sigma, 1)$,

$$\tilde{\gamma} = \frac{\sigma}{\sigma + (1 - \sigma)\tau(\sigma, 1)},$$

constitutes an upper bound for the equilibrium parameter $\bar{\gamma}$. Consider then the pair (μ, p) satisfying the following equations:

$$u_{l,L}^* - u_{l-1,L}^* = \psi(\mu)\left(p - c_L - u_{l-1,L}^*\right), \tag{A.10}$$

$$\eta(\mu)\left(\tilde{\gamma} v_L + (1 - \tilde{\gamma}) v_H - p\right) = k. \tag{A.11}$$

The (two) solutions of these equations represent the intersections between the L-type indifference curve associated to the l-th application in the observable type case and the firms' isoprofit curve associated to markets with effective composition $\tilde{\gamma}$. Considering the solution with the higher value of p, it is immediate to verify that this value is increasing in v_H. Let value v_H^1 be the value of v_H such that this solution equals $p = v_L - k$. For all $v_H > v_H^1$, there can then be no intersection between an indifference curve passing through the point (p, μ), determined by (A.10-A.11), and the isoprofit curve Π_L. Since for any $N > l$, the equilibrium parameter $\bar{\gamma}$ is lower than $\tilde{\gamma}$ (there are fewer low types), the same property holds for all indifference curves passing through the equilibrium pooling market.

We then set $\bar{v}_H = \max\{v_H^0, v_H^1\}$. $\square$

A.3 Proof of Proposition 3

The argument for the case $c_H \geq v_L - k$ is in the main text. We thus consider $c_H < v_L - k$ and $k \to 0$.

Case $N = 1$**.** The L-type's utility in the limit as $k \to 0$ is

$$u_{1,L} = v_L - c_L,$$

as can be seen from (9)-(10). The associated indifference curve is

$$I_L(c_L, v_L - k) = \left\{(p, \mu) : v_L - c_L = \frac{1 - e^{-\mu}}{\mu}(p - c_L)\right\}.$$

The H-type market in the limit allocation is determined by the intersection of this indifference curve with the vertical isoprofit curve $p = v_H$. The H-types' limit trading probability is thus given by

$$\psi_{1,H} = \frac{v_L - c_L}{v_H - c_L},$$

and their limit market utility is

$$u_H(1) = \frac{(v_L - c_L)(v_H - c_H)}{v_H - c_L}.$$

Case $N = 2$**.** Since $\lim_{k \to 0} u_{L,1} + c_L = v_L > c_H$, we have $l = 1$. Hence, for $N = 2$, there is a partial pooling equilibrium, as described in Proposition 2. Let $\bar{\gamma}$ be the equilibrium proportion of low types in the mixed market. In the limit as $k \to 0$, the trading probability in the mixed market, denoted by $\bar{\psi}$, is determined by the intersection of the L-type's indifference curve and the vertical isoprofit-curve $p = \bar{\gamma} v_L + (1 - \bar{\gamma}) v_H$:

$$\bar{\psi} = \frac{v_L - c_L}{\bar{\gamma} v_L + (1 - \bar{\gamma}) v_H - c_L}.$$

The associated equilibrium payoff for the L-type as a function of $\bar{\gamma}$ is then

$$\begin{aligned} u_{2,L} &= \bar{\psi}(\bar{\gamma} v_L + (1 - \bar{\gamma}) v_H - c_L) + (1 - \bar{\psi})(v_L - c_L) \\ &= \left(1 + \frac{(1 - \bar{\gamma})(v_H - v_L)}{(1 - \bar{\gamma})(v_H - v_L) + (v_L - c_L)}\right)(v_L - c_L). \end{aligned}$$

The limit trading probability in the H-market, $\psi_{2,H}$, makes the L-type indifferent between sending his second application to the mixed market and the H-market. It is thus determined by

$$\begin{aligned}
&\left(1+\frac{(1-\bar{\gamma})(v_H-v_L)}{(1-\bar{\gamma})(v_H-v_L)+(v_L-c_L)}\right)(v_L-c_L)=\psi_{2,H}(v_H-c_L)+(1-\psi_{2,H})(v_L-c_L)\\
\Leftrightarrow\quad &\psi_{2,H}=\frac{(1-\bar{\gamma})(v_L-c_L)}{(1-\bar{\gamma})(v_H-v_L)+v_L-c_L}.
\end{aligned}$$

The H-type's market utility in the limit as $k\to 0$ is then

$$u_{2,H}=\psi_{2,H}(v_H-c_H)+(1-\psi_{2,H})\bar{\psi}(\bar{\gamma}v_L+(1-\bar{\gamma})v_H-c_H). \tag{A.12}$$

Note that the trading probability in the H-market, $\psi_{2,H}$, is strictly decreasing in the equilibrium composition $\bar{\gamma}$. That is, the higher the proportion of low types in the mixed market, the smaller is the probability for high types to trade in the H-market. The H-type's market utility is in turn increasing in $\psi_{2,H}$ and decreasing in $\bar{\gamma}$. Hence, $u_{2,H}$ is strictly decreasing in $\bar{\gamma}$.

Fixed point. The endogenous composition of the mixed market in the limit as $k\to 0$ is determined by the equations

$$\begin{aligned}
\bar{\gamma} &= \frac{\sigma}{\sigma+(1-\sigma)(1-\psi_{2,H})},\\
\psi_{2,H} &= \frac{(1-\bar{\gamma})(v_L-c_L)}{(1-\bar{\gamma})(v_H-v_L)+v_L-c_L}.
\end{aligned}$$

The first equation can be written as

$$\bar{\gamma}\sigma+\bar{\gamma}(1-\sigma)(1-\psi_{2,H})=\sigma\quad\Leftrightarrow\quad\frac{\bar{\gamma}}{1-\bar{\gamma}}(1-\psi_{2,H})=\frac{\sigma}{1-\sigma}.$$

Consider now an increase in σ. This strictly increases the RHS of the last equation, hence the LHS must increase as well. Recalling that $\psi_{2,H}$ is strictly decreasing in $\bar{\gamma}$ (so $1-\psi_{2,H}$ is strictly increasing), this requires that $\bar{\gamma}$ increases. Hence, the composition of the mixed market $\bar{\gamma}$ strictly increases in the population parameter σ with the following limits.

- As $\sigma\to 1$, $\bar{\gamma}\to 1$, $\bar{\psi}\to 1$ and $\psi_{2,H}\to 0$.
- As $\sigma\to 0$, $\bar{\gamma}\to 0$ and $\bar{\psi},\psi_{2,H}\to\frac{v_L-c_L}{v_H-c_L}=\psi_{1,H}$.

We denote by $u_H(2)$ the limit market utility (A.12) with $\bar{\gamma}$ and $\psi_{2,H}$ as the fixed point of the equations above.

Welfare comparison. Having established the limit allocation for $k\to 0$, we want to argue that for any k sufficiently small, there is a threshold for $\bar{\sigma}_k$ such that $u_H(1)<u_H(2)$ if $\sigma<\bar{\sigma}_k$

and $u_H(1) > u_H(2)$ if $\sigma > \bar{\sigma}_k$. We know that the market utility $u_H(1)$ is independent of σ and already showed that, in the limit case $k \to 0$, $u_H(2)$ is strictly decreasing in $\bar{\gamma}$, which in turn is strictly increasing in σ. Hence, the limit market utility $u_{2,H}$ is strictly decreasing in σ. We thus have a threshold $\bar{\sigma} \in [0, 1]$ such that the limit values of H-types' market utilities satisfy $u_H(1) < u_H(2)$ if $\sigma < \bar{\sigma}$ and $u_H(1) > u_H(2)$ if $\sigma > \bar{\sigma}$. What remains to be shown is that this threshold lies on the interior of the unit interval. This follows from

$$\begin{aligned}\lim_{\sigma \to 0}(u_H(2) - u_H(1)) &= (1 - (1 - \psi_{1,H})^2)(v_H - c_H) - \psi_{1,H}(v_H - c_H) \\ &= \psi_{1,H}(1 - \psi_{1,H})(v_H - c_H) > 0,\end{aligned}$$

and

$$\begin{aligned}\lim_{\sigma \to 1}(u_H(2) - u_H(1)) &= (v_L - c_H) - \frac{(v_L - c_L)(v_H - c_H)}{v_H - c_L} \\ &= -\frac{(v_H - v_L)(c_H - c_L)}{v_H - c_L} < 0.\end{aligned}$$

□

A.4 Proof of Proposition 4

A straightforward extension of Proposition 6 in Kircher (2009) shows that $\lim_{N\to\infty} u^*_{N,L} = v_L - c_L - k$. We now want to prove that the probability with which the H-type is hired in equilibrium tends to zero. Since wages are bounded above by the firms' valuation (net of entry cost), this directly implies $\lim_{N\to\infty} u_{N,H} = 0$. Letting $(\mu_{1,H}(N), p_{1,H}(N))$ describe the terms of trade in market $(1, H)$ when the number of available applications is N, we can define the probability of being hired when sending $\tilde{N}$ applications to market $(1, H)$:

$$\alpha(\tilde{N}, N) := 1 - \left(1 - \left(1 - e^{-\mu_{1,H}(N)}\right) / \mu_{1,H}(N)\right)^{\tilde{N}}.$$

Since $\mu_{1,H} \leq \mu_{n,H}$ for all n, $\alpha(N, N)$ is an upper bound for the equilibrium probability with which the H-type is hired when sending N applications.

Now suppose each worker has available $2n + j$ applications where $n \in \mathbb{N}$ and $j \in \{0, 1\}$. If the L-type sends no applications to any of the H-type markets, his payoff is $u^*_{2n+i,L} < v_L - c_L - k$. If instead he sends $n + j$ applications to the L-markets with the lowest $n + j$ wages and n applications to market $(1, H)$, his payoff is

$$\alpha(n, 2n + j)(p_{1,H}(2n + j) - c_L) + (1 - \alpha(n, 2n + j))u^*_{n+j,L}. \quad \text{(A.13)}$$

In equilibrium, (A.13) must be smaller than $v_L - c_L - k$. Since $\lim_{n\to\infty} u^*_{n+j,L} = v_L - c_L - k$ and $p_{1,H}(2n+j) - c_L > c_H - c_L > v_L - c_L - k$, this requires $\lim_{n\to\infty} \alpha(n, 2n+j) = 0$.

Finally, we want to show that $\alpha(n, 2n+j) \to 0$ implies $\alpha(2n+j, 2n+j) \to 0$. To this end, notice that the function $\alpha(\cdot, 2n+j) : \mathbb{R} \to [0,1]$ is strictly increasing and strictly concave with $\alpha(0, 2n+j) = 0$. Hence,

$$\alpha(n, 2n+j) > \frac{n}{2n+j}\alpha(2n+j, 2n+j).$$

Given $\lim_{N\to\infty} n/(2n+j) = 1/2$, this inequality and the property $\lim_{n\to\infty} \alpha(n, 2n+j) = 0$ imply $\lim_{n\to\infty} \alpha(2n+j, 2n+j)/2 = 0$. Hence, $\lim_{N\to\infty} \alpha(N, N) = 0$. As we stated above, $\alpha(N, N)$ is an upper bound for the equilibrium probability with which H-type workers are hired. In the limit this type is hired with probability zero and $\lim_{N\to+\infty} u_{N,H} = 0$. □

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