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Auctioning Long-Term Projects under Financial Constraints

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We consider a procurement auction for the provision of a basic service to which an add-on must later be appended. Potential providers are symmetric, have private information on their cost for the basic service and the winning firm must also implement the add-on. To finance value-enhancing activities related to the add-on, this firm may need extra funding by outside financiers. Non-verifiable effort related to these activities creates a moral hazard problem which makes the firm's payoff function for the second period concave in returns over the relevant range. Concavity has two effects. First, it makes it more attractive to backload payments to facilitate information revelation. Second, uncertainty on the cost of the add-on introduces a background risk which requires a risk premium. In this context, we characterize the optimal intertemporal structure of payments to the winning firm, equilibrium bidding behavior and reserve prices for a first-price auction.

Key words: Auctions, Procurement, Financial constraints, Dynamic mechanism design, Asymmetric information, Uncertainty, Endogenous risk aversion.

1. INTRODUCTION

MOTIVATION. In many countries throughout the world, service provision in the public sector is decided by means of some sort of competitive bidding. To illustrate, in the European Union, the core principles for public procurement aim to establish a playing field for open competition.[1] Yet, at the time of tendering for the provision of long-term durable projects, bidders as well as the government agency in charge may not be aware of all the specific needs and costs that future stages in the provision of services may require. For instance, when the Norwegian Road Administration (*Statens vegvesen*) renovates and upgrades an existing infrastructure, it oftentimes requires additional, technically different and innovative work from its contractors.[2] *Ex ante*, bidding is thus surrounded by significant uncertainty on what will be the future contours of the project. *Ex post*, the

1. In the EU, open procedures, and more specifically first-price or first-score auctions which are open to any qualified bidder, constitute 73% of all tenders announced in the Official Journal (PWC, 2011).

2. Examples include road construction and maintenance that has led to necessary work on water and sewage lines below the construction site being included in the initial contract (KOFA cases 2016/1 and 2016/148). Other relevant examples include unexpected additional work during renovation (KOFA case 2014/14) and the extension of service provision to additional market segments (KOFA cases 2020/193 and 2021/54).

public agency is stuck in a bilateral relationship with the firm having won the bidding stage. This scenario comes with mixed blessings. On the one hand, that competitive pressure is mute at later stages might be a source of extra rent for the winning firm; a force that could *a priori* call for extra payments. On the other hand, if this procurment agency has not committed enough funds to the project over its whole life cycle, for instance because raising and committing public funds is increasingly more costly over time, the winning firm might have to approach outside financiers for additional financing of the investment needed at these stages. Thus, part of this extra rent might be dissipated through agency frictions on financial markets.

Examples of contract adjustments in public procurement abound even though they are also often viewed with an eye of caution as they are expected to significantly deflate value for money. Pieces of casual evidence suggest that the debate on how the provision of add-ons should be regulated in the public sector remains by and large unsettled. On the one hand, motivated by a surge in the scope and frequency of contract adjustments in procurement contexts, the most recent EU directives on public procurement, utilities procurement and concessions open the door to considering contract adjustments in future public contracts with less reluctance than in the past. On the other hand, the case of highways in France illustrates how a more cautious stance might have been implemented in practice. One of the key missions of the French Autorité de Régulation des Transports (ART) is indeed to evaluate the relevance of any extra clause that might be appended to concession contracts as soon as those changes may have a significant impact on tariffs and concession length.

This paper offers a theoretical framework that informs what should be best practices in those environments. It thus contributes to deepening our understanding of the effects of add-ons and the design of public policy. In a nutshell, our analysis shows how firms' incentives and bidding behavior respond to the intertemporal profile of payments for basic services, the magnitude of the possible agency costs that pertain to the contractual relationship with outside financiers involved in an add-on and the degree of competition surrounding the tender.

Model and Main Insights. A public agency wants to procure a basic service through a competitive tender. In line with public procurement law and practices in many countries (see for instance, Branzoli and Decarolis (2015), Decarolis (2018) and Hyytinen et al. (2018)), a first-price auction is used.[3] To this long-lasting basic service whose contours are well known at the time of bidding, an add-on whose costs remain *ex ante* uncertain will be added later on. At the time of bidding, potential contractors are privately informed about their production costs for the well-defined long-term basic service. Later on, when the exact specifications of the add-on are revealed, information regarding the corresponding costs becomes available. To reflect existing practices, the task of completing the project with the add-on only accrues to the winning bidder who is already in charge of the basic service. Reasons behind such bilateral lock-in are that various transaction and administrative costs may preclude opening further rounds of bidding, or that, because there exist strong technological complementarities between this add-on and the basic service, sourcing from other firms is *de facto* impossible or inefficient.

3. In a more complex setting, the tender procedures might be multidimensional and also take into account different quality dimensions, but a winning bidder would still be asked to provide the basic service based on its proposal and at the price stipulated in the winning bid.

Importantly, returns on the basic service may not suffice to cover the investment needed to implement the add-on efficiently. The firm may have to rely on outside financiers to take a share of that investment. In return, those financiers take a share of the profit that such an add-on generates. Because the firm is not fully paid for its effort in innovating on the quality of the add-on, there is moral hazard in the contractual relationship with financiers. The magnitude of this moral hazard problem depends on how much profit made on the basic service can be used as collateral. Because agency frictions on the financial market decrease with that profit and do so at an increasing rate, the firm *in fine* inherits an endogenous utility function that is concave in its second-period payoff, at least over the range where outside financiers are ready to contribute. Concavity of a firm's payoff function thus endogenously follows from agency frictions that the firm faces when raising outside finance to undertake the additional investment needed to increase efficiency in the provision of the add-on.

This concavity unveils a new *Income Effect* that is absent from more standard analysis of dynamic procurement contracts which most often assume quasi-linear payoffs. This effect has far-reaching consequences. Under complete information, the optimal split between the public and private money needed to finance the add-on is completely determined by a simple rule: The firm's marginal utility of income must equal the cost of public funds. This optimal distribution between public and private funding of the add-on is an important benchmark to assess the role of asymmetric information. Under asymmetric information, each potential contractor indeed exaggerates its costs for the basic service not only with a view on consequences for its intertemporal profits for this long-term service but also with an eye on how such a manipulation might affect agency frictions with outside financiers. Because of concavity in the second-period payoff, the marginal utility of such a manipulation is higher at lower profit levels. The firm's benefit from exaggerating costs on the basic service also decreases as the firm obtains a greater return for providing the basic service. This pushes the public agency to backload payments for the basic service so that incentive compatibility constraints are relaxed. In turn, a public commitment to grant more profit for the basic service also facilitates the firm's access to financial markets. The moral hazard problem with outside financiers is thus of a lower magnitude and innovation related to the add-on becomes more likely. Finally, this *Income Effect* and the associated backloading of payments lead firms to bid more aggressively in the tender.

Consider now the case where the cost of the add-on remains the firm's private information. Now a *Risk Effect* is also at play. This effect raises the marginal utility of income in the second period and exacerbates incentives to manipulate the cost of the basic service. The argument for why it is so goes as follows: To ensure second-period incentive compatibility, the public agency offers the firm a constant price for the add-on independently of its cost. The firm's profit from the add-on then fluctuates with this cost realization which amounts to having the firm bear an endogenous background risk. Because the firm's second-period payoff is concave, this extra risk increases agency frictions. The direct consequence of such background risk is that an additional risk premium must be paid to ensure the firm's participation. A second, less obvious, consequence is that the firm's marginal utility of income might also increase with that risk. The firm thus finds it more attractive to exaggerate its cost for the basic service. Fighting this effect has two important consequences. First, it calls for backloading even more of the payment for the basic service to the second period. Second, inducing participation now being more costly, the public agency must raise the reserve price in the auction in comparison with a scenario of complete information on the cost of the add-on. This suggests that long-term projects with add-ons surrounded with much uncertainty and asymmetric information should be under a hard budget constraint.

Literature Review. This paper follows and builds on an approach of dynamic procurement developed in a companion paper, Arve and Martimort (2016). As in that former paper, we are interested in the impact of a concave utility function defined over second-period payoffs on intertemporal pricing. While in Arve and Martimort (2016), we take this utility function as given and invoke existing financial constraints as a motivation, the analysis of the current paper derives those constraints from agency considerations on financial markets; thereby endogenizing the utility function from first principles. Beyond stressing the role of a proper design of the intertemporal profile of payments to reduce the winning firm's information rent, which, even with the introduction of competition, is similar to that in Arve and Martimort (2016) and echoing similar well-established results in the literature (Myerson (1981), Riley and Samuelson (1981), McAfee and McMillan (1986), Riordan and Sappington (1987), Laffont and Tirole (1987), among others), the current model allows for screening on the extensive margin by means of a reserve price. Instead, Arve and Martimort (2016)'s focus is on screening on the intensive margin and, more generally, discuss how risk aversion in a dynamic procurement environment with uncertainty would affect output of a single firm. In sharp contrast, the current paper analyzes a scenario with competition for a fixed-size basic service and a fixed-size add-on. To improve screening, the public agency can hereafter also use a reserve price in the bidding process so as to reduce participation, an extreme form of rent extraction.

There is also a small literature on contractual design in static procurement contexts when agents are risk averse (Salanié (1990), Laffont and Rochet (1998) and Bontems and Thomas (2003)). We depart from this literature in two respects. First, we endogenize the degree of risk aversion from underlying financial constraints. Second, our model is dynamic and the fact that risk neutrality prevails in the first period significantly simplifies technicalities.

Our paper is also related to a much broader literature on single-unit static auctions with risk-averse bidders (Holt (1980), Riley and Samuelson (1981), Maskin and Riley (1984), Matthews (1987)). These papers compare standard auction formats under risk aversion and show that the Revenue Equivalence Theorem (Myerson (1981), Riley and Samuelson (1981)) fails. We depart from this literature by embedding risk considerations in a dynamic context and by endogenizing the concave utility function from agency considerations on financial markets. More recently, the literature on risk aversion in auctions has analyzed the choice of an optimal reserve price (Hu et al. (2010), Hu (2011)) and various asymmetries between bidders (asymmetric valuations in Menicucci (2003) and risk attitudes in Maréchal and Morand (2011)). Finally, Esö and White (2004) focus on how an *ex post* background risk impacts bidding behavior. They show how bidders who exhibit decreasing absolute risk aversion engage in precautionary bidding and, in a common value environment, shade their bids by more than the corresponding risk premium. A similar bid reduction is present in our independent private value setting, but is not always dominating. Another important modeling difference comes from the fact that the background risk that pertains to the add-on is endogenously derived from our assumption that bidders have private information on their cost of providing this add-on.

By focusing on an environment with an uncertain component, our model bears some similarity to McAfee and McMillan (1986) who look at the optimal contract in an environment where part of the cost is unknown *ex ante*. However, these authors limit the analysis to linear contracts and, more importantly, their focus is on the trade-off between ex post screening and moral hazard in a static environment, not on the dynamic payment structure and bidding behavior in a dynamic setting. A similar trade-off between *ex ante* competition and *ex post* moral hazard is also analyzed in Chakraborty et al. (2021).

In this paper, we abstract from the question of pre-project planning (see for instance Krähmer and Strausz (2011)) that could potentially reduce uncertainty on future add-ons. One of our concerns is instead how a given level of uncertainty affects contracting and bidding behavior. For the same reason, we also differ from the part of the literature that studies the auctioneer's incentives to disclose information in private value settings (Ganuza (2004), Board (2009), Ganuza and Penalva (2010), Ganuza and Penalva (2019)) as well as from papers showing how a simple initial design may be optimal even if it implies cost overruns and contract modifications (Ganuza (2007), Herweg and Schwarz (2018)).

On the empirical front, a line of research has shown the impact of contract adjustments and change orders on prices and bids. Bajari et al. (2014) estimate adaptation costs in paving projects in California to be 8-14% of the winning bid. Jung et al. (2019) show, using construction data from Vermont, that markups are higher in auctions that entail renegotiated tasks. De Silva et al. (2017) look at how project modifications impact bidders' costs. In this paper, we consider contract adjustments in an optimal contracting model, and ask how contract adjustments in the form of an additional risky task affects bidding behavior and how public authorities should respond in designing auctions and contracts.

At the source of the agency frictions with outside financiers that underlies our analysis is a moral hazard problem. The firm in charge of developing the add-on may exert some effort to enhance the value of the add-on. The analysis of innovation has been an active front in the procurement literature over recent years. While Che et al. (2021) study a value-enhancing innovation, Tan (1992), Piccione and Tan (1996), Bag (1997) and Arozamena and Cantillon (2004) have studied cost-reducing R&D in procurement. Our framework is sufficiently general to include both cost-reducing innovations and more general value-enhancing innovations. However, these papers differ from ours in terms of the timing of the investment. Investment takes place before competition in those papers, while it happens after in our analysis. Additionally, we derive an endogenous utility function that determines strategic bidding behavior from this moral hazard problem.

Organization. The rest of the paper is organized as follows. The model is presented in Section 2. Section 3 provides micro-foundations for the firm's utility function on second-period payoffs. In that section, we model in detail the moral hazard problem vis-à-vis outside financiers that allows us to derive the shape of this utility function. Equipped with this specification, Section 4 presents the complete information benchmark for our main model. We derive the optimal allocation of public and private funds to finance the add-on. Moving to the scenario of asymmetric information, we characterize incentive-feasibility conditions in Section 5. The optimal contract for the basic service, equilibrium bidding strategies and the optimal reserve price are presented in Section 6. Section 7 investigates how our results are modified when there is also asymmetric information on the cost of the add-on. Section 8 proposes alleys for future research. Proofs are relegated to the Appendix.

2. THE MODEL

We consider an environment where several firms compete for the provision of services in a public procurement context. A public agency (henceforth *the principal*) organizes an auction to select and contract with one of those $n+1$ firms ($n \geq 0$) for the provision of a basic service. This basic service is durable and is to be provided over two periods. In the second period, an add-on is also required from the winning bidder.

To simplify the modeling of the demand side, we assume that the size of these two services is fixed and normalized to one unit.[4] Uncertainty around the cost of the add-on puts the selected firm's future returns at risk. We are interested in the impact of this risk first, on the firms' bidding behavior and, second, on the intertemporal structure of the contract awarded to the winning firm. In a separate section, we provide micro-foundations for the firm's endogenous risk aversion by assuming that to enhance the value of the add-on, the firm may innovate. To this end, the firm must undertake a costly investment. To finance this outlay, the firm might need to approach outside financiers if the basic service does not provide enough returns. The contractual relationship with those outside financiers is plagued with moral hazard and agency costs. The main analysis focuses on the impact of these agency costs on the structure of the optimal procurement contract.

Technology, contracts and information. The basic service yields a gross surplus S_1 in each period. The winning firm provides this service at a cost equal to θ_1. The gross surplus from providing the add-on for the second period is S_2 and the firm's cost of doing so, absent any innovation, is θ_2.

The service provider is selected by means of a first-price auction with a reserve price. The principal offers a contract, consisting of payment specifications for the basic service over the two periods as a function of the winning firm's cost (announcement) as well as a menu of prices for the add-on. The payments for the basic service are denoted by $b(\theta_1)$ and $b(\theta_1)+y(\theta_1)$ for periods 1 and 2 respectively. The second-period premium $y(\theta_1)$ – which may also be negative – allows for a possible non-stationarity of payments for this service. The second-period payment for the add-on is denoted by $p(\theta_1,\theta_2)$. The exact specifications required for the add-on are not completely known *ex ante* by the contracting parties and the principal therefore might *a priori* offer a menu of prices for this component, one for each state of the world that might realize later on.

At the time of the tender, each firm i has private information on its cost θ_{1i} of providing the basic service.[5] These cost parameters are independently drawn across firms from a common knowledge and atomless cumulative distribution F with an everywhere positive density f whose support is $\Theta_1 = [\underline{\theta}_1, \overline{\theta}_1]$. To ensure that the optimization problems below are regular, we sometimes assume that $\overline{\theta}_1 - \underline{\theta}_1$ is not too large and

4. In the examples mentioned in Footnote 2, this means that the scope of the upgrade or renovation work is not up for discussion and the extension of the procurement contract to new segments or additional work is also not of variable size.

5. We will ignore the subscript i whenever possible.

also adopt the standard *Monotone Hazard Rate Property*:[6]

$$\frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right) \geq 0 \text{ for all } \theta_1 \in \Theta_1. \tag{2.1}$$

Firms are also symmetric in terms of the distribution of their innate cost of providing the add-on. To capture the idea that the add-on is not completely defined at the time of contracting, we assume that its cost is uncertain at this stage. *Ex ante*, there is symmetric but incomplete information on the cost parameter θ_2. Once this cost has realized, that cost may be common knowledge (main scenario investigated in Sections 4-6) or the firm may have private information on it (a scenario investigated in Section 7). To maintain a tractable analysis, we suppose that θ_2 is drawn from a common knowledge distribution on a discrete support $\Theta_2 = \{\underline{\theta}_2, \overline{\theta}_2\}$ with respective probabilities ν and $1-\nu$, where $\nu \in (0,1)$. We denote $\Delta = \overline{\theta}_2 - \underline{\theta}_2 > 0$. There is no correlation across the costs θ_1 and θ_2 and, more generally, there is no technological linkage across services.[7] On the benefit side, we also suppose that S_2 is large enough to ensure that this add-on is always valuable.[8]

Preferences. Denoting respectively by $u_1(\theta_1) = b(\theta_1) - \theta_1$ and $\pi(\theta_1) = b(\theta_1) + y(\theta_1) - \theta_1$ the firm's first-period and second-period profit from the basic service, and by $u_2(\theta_1, \theta_2) = p(\theta_1, \theta_2) - \theta_2$ its second-period profit from the add-on, the principal's intertemporal payoff can be written as

$$\mathcal{S}(\theta_1) - (1-\beta)u_1(\theta_1) - \beta(1+\tau)\left(\pi(\theta_1) + \mathbb{E}_{\theta_2}(u_2(\theta_1, \theta_2))\right), \tag{2.2}$$

where $1-\beta$ and β are the relative (accounting) weights on the first and second period respectively, τ is the cost of second-period public funds and the social value of the projects (without any innovation) is

$$\mathcal{S}(\theta_1) = S_1 - (1+\beta\tau)\theta_1 + \beta(S_2 - (1+\tau)\mathbb{E}_{\theta_2}(\theta_2)).$$

The expression of the principal's objective in (2.2) highlights the rent-efficiency trade-off that characterizes contracting under informational asymmetries. The principal cares about the social value of the project but would also like to minimize the share of that surplus that accrues to the firm. Of course, this maximization problem should still induce the firm's participation in the tender mechanism and be incentive compatible when asymmetric information is a concern.[9]

The fact that there exists a cost of public funds τ for the second period means that second-period payments are more costly to the principal than payments made earlier on.

6. Bagnoli and Bergstrom (2005).

7. This assumption stands in contrast with the literature on public-private partnerships (PPPs). PPP environments often include externalities across tasks, such as effort in a first stage that reduces costs in the second stage (Hart et al. (1997), Hart (2003), Bennett and Iossa (2006), Schmitz (2005), Martimort and Pouyet (2008) and Iossa and Martimort (2012, 2015)). Contrary to our focus on two perfectly complementary tasks (basic service and add-on) that, by assumption, are executed within the same contract, this literature asks whether those tasks should be bundled or not.

8. A review by the Swedish National Audit Office (Riksrevisionen, 2021) shows that it is extremely uncommon for a project to be cancelled at the add-on stage. This is perhaps best illustrated by a quote from a municipality in Sweden where extensive additional work was required for the construction of a bridge and where the attitude was *"better with an expensive bridge than no bridge at all"* (authors' own translation from Swedish of KPMG (2021)).

9. We come back to those incentive-feasibility constraints in Section 5.

Indeed, although second-period compensations may have been planned well in advance, the need to channel extra public funds towards the project may require giving up other valuable projects or levy more funds elsewhere in the economy at an increasingly higher marginal cost. This cost of public funds plays a key role in assessing the extent to which outside financiers are involved in the financing of the investment.

Studying how second-period financial constraints impact incentives requires to move away from more standard models of procurement and endogenize the firm's risk preferences with respect to second-period uncertain returns. Still, we shall maintain the more standard assumption that firms are risk neutral with respect to their first-period returns. Accordingly, we shall express the winning firm's intertemporal payoff as

$$(1-\beta)u_1(\theta_1)+\beta\mathbb{E}_{\theta_2}(v(\pi(\theta_1)+u_2(\theta_1,\theta_2))),$$

where the firm's utility function v defined on second-period returns is increasing and concave ($v'>0$, $v''\leq 0$) over the relevant range. For the sake of this section, we shall view v as being exogenous. Section 3 provides sound micro-foundations for those preferences.

That the firm's second-period marginal utility is non-increasing captures the fact that, even though the firm might face costly access to the capital market when raising outside funds to finance the add-on, the marginal cost of such outside financing decreases when the firm's returns from the basic service are of greater magnitude. Intuitively, the firm can use more of those returns as collateral which helps attract outside financiers. *A contrario,* that the firm remains risk neutral with respect to its first-period returns captures the idea that returns on the basic service are well known, stable enough and do not require any major investment, or that any such investment has already been redeemed. In any case, outside finance is only called for to finance the add-on.

Auction and Contract Design. The principal runs a first-price auction and commits to a long-term contract with the winning bidder. That contract regulates prices for the basic service and the add-on over both periods. In such an auction, potential service providers thus bid for a complex object; a long-term contract. By means of such a contract, the principal may choose the intertemporal structure of payments so as to reduce its total cost of provision. In particular, the payment for the basic service may be spread over time; a possibility which certainly has some appeal when the service provider is risk averse and future profits from the add-on remain uncertain at the time of contracting.

From the *Revelation Principle* in a dynamic context (Baron and Besanko (1984), Battaglini (2005), Pavan et al. (2014), among others), a long-term contract awarded to the lowest-cost bidder can be viewed as a direct revelation mechanism that stipulates payments in each period as a function of the firm's report of its cost for the basic service as well as a menu of prices for the add-on. Notice that we should allow the price of the add-on to be a function not only of the announced cost for the basic service but also of the second-period announcement of the cost for the add-on if the firm has private information on it. Thus, a mechanism is of the form $\left\{b(\hat{\theta}_1),y(\hat{\theta}_1),p(\hat{\theta}_1,\hat{\theta}_2)\right\}_{\hat{\theta}_1\in\Theta_1,\hat{\theta}_2\in\Theta_2}$ where $\hat{\theta}_1$ is the firm's announcement of its cost for the basic service and $\hat{\theta}_2$ that of its cost for the add-on. These reports will of course be truthful in equilibrium.

The winning bidder makes the lowest announcement $\hat{\theta}_1$ for the basic service. Provided that the payment schedule $b(\cdot)$ is increasing, everything happens as if potential contractors were bidding for the basic service with the winning bidder expressing the lowest cost for the basic service. In turn, this bid $b(\hat{\theta}_1)$ determines the second-period additional payment $y(b^{-1}(b(\hat{\theta}_1)))$ and the menu of prices for the add-on $\{p(b^{-1}(b(\hat{\theta}_1)),\hat{\theta}_2)\}_{\hat{\theta}_2\in\Theta_2}$.

Timing. The auction cum contracting game unfolds as follows:

1. Firm i (for $i=1,...,n+1$) privately learns its cost parameter θ_{1i} for the basic service.
2. The principal announces the rules of the first-price auction, i.e., a reserve price as well as a long-term contract for the winning firm. This contract stipulates the payments for the basic service in both periods and for the add-on in the second period.
3. Firm i decides whether to participate or not. If it does, it announces its cost $\hat{\theta}_{1i}$. The lowest-announcement firm, $i_0=\min_i\hat{\theta}_{1i}$, is awarded the contract.[10]
4. The first payment for the basic service $b(\hat{\theta}_{1i_0})$ is made to the winning firm i_0 upon delivery of the basic service for the first period.
5. The second payment for the basic service $b(\hat{\theta}_{1i_0})+y(\hat{\theta}_{1i_0})$ is made to the winning firm i_0 upon delivery of the basic service for the second period.
6. The winning firm learns the value of the cost for the add-on, θ_{2i_0}. So does the principal in the scenario where this cost is common knowledge.
7. The winning firm invests I to innovate and possibly enhance the value of the add-on. To do so, this firm approaches outside financiers. This financing stage of the game is further developed in Section 3 below.
8. If privately informed on the cost of the add-on, this winning firm then announces $\hat{\theta}_{2i_0}$. This firm provides the add-on at a price $p(\hat{\theta}_{1i_0},\hat{\theta}_{2i_0})$ stipulated by the contract.

This multi-stage Bayesian game is solved by backward induction. We normalize, without loss of generality, reservation payoffs for all parties to zero.

3. MORAL HAZARD, FINANCIAL CONSTRAINTS AND ENDOGENOUS RISK AVERSION

In this section, we provide micro-foundations for the second-period preferences v. This step is important to foster our understanding of how private and public funds are jointly used to finance long-term projects and to deepen the interpretation of our findings.

We first consider a one-shot model in which a firm can invest in a value-enhancing innovation. The firm enjoys a safe income π that can be pledged as collateral.[11] Investing I may generate an extra benefit $\gamma>0$ to the firm with probability $e\in[0,1]$, where e is its non-verifiable effort. To always ensure interior solutions, we assume that the firm's disutility of effort $\psi(e)$ satisfies $\psi'(e)\geq 0$, $\psi''(e)>0$, $\psi'''(e)\geq 0$ and the Inada conditions $\psi(0)=\psi'(0)=0$ and $\psi'(1)=+\infty$. For future reference, the first-best effort level e^{fb} that maximizes the expected value of the innovation $\gamma e-\psi(e)$ solves $\psi'(e^{fb})=\gamma$.

10. In case of a tie among several bidders (a zero-probability event given that the cost distribution has no atoms), the winning firm is randomly selected with equal probabilities.

11. In our main model, π can be viewed as profits from the basic service and a standard add-on.

There is a competitive market of financiers[12] who have expertise to evaluate pledgeable profits[13] and can observe whether an innovation is successful.[14] A financial contract thus stipulates payments $\overline{T}$ and $\underline{T}$ depending on whether an innovation occurs or not.

We will further assume that the investment is valuable even when agency considerations arise and effort is reduced to the level e^m that would be chosen by a monopolistic lender when financing a firm requiring a liability rent $R(e)$ for effort e. This condition writes as

$$I \leq \gamma e^m - \psi(e^m) \text{ where } \psi'(e^m) + R'(e^m) = \gamma. \tag{3.3}$$

As formally shown in Appendix A, moral hazard, together with limited liability, is the source of agency costs of outside finance. To exert effort, the firm must receive a liability rent $R(e) = e\psi'(e) - \psi(e)$.[15] Outside financiers need to cover not only the net investment outlay $I - \pi$ but also the liability rent $R(e)$ required to induce an effort e. Implementing a second-best effort $e^{sb}(\pi)$ below the first-best level e^{fb} reduces this rent and thus eases the financial constraint. Of course, if the profit level π is too low, the innovation cannot be financed.

A higher pledgeable profit π facilitates access to outside finance. The expected benefit of an innovation is now enough to cover not only a lower net outlay $I - \pi$ but also a greater liability rent for the firm. Since inducing more effort requires giving up more of this rent, less effort distortions are now needed and the expected benefit of an innovation increases. Over the range where this financial constraint matters and a second-best effort $e^{sb}(\pi)$ is implemented, increasing π thus increases the firm's payoff at a rate more than unity. Formally, $v'(\pi) > 1$ for $\pi \in (\hat{\pi}, I)$ where $\hat{\pi}$ defines a lower bound below which outside finance is not feasible.[16]

For $\pi \geq I$, the innovation is self-financed. Effort is set at its first-best level and the firm appropriates the whole expected benefit from the innovation. When π increases towards I, agency costs decrease but at a decreasing rate. Indeed, as effort gets closer to the first-best, a slight increase in the pledgeable profit π also slightly increases effort,

12. Assuming that financiers are competitive means that the firm ends up enjoying all gains from innovation up to the agency frictions coming with outside finance. It makes the comparison with a self-financed innovation straightforward.

13. That outside financiers have expertise not found in the public administration is sometimes viewed as a justification for bringing outside financiers into the development of major public-private partnerships. See Iossa and Martimort (2011, 2012, 2015) for some modelling on those issues.

14. Observability is consistent with our main analysis (Sections 4-6), where the realization of the cost of the add-on is public information. Furthermore, innovations in the road industry, such as *"the use of improved materials for the persistence of road building roads for example recycled materials, solar roads, foamed bitumen, information technology innovation that comprises the practice of improved technologies"* (Oad et al. (2001)), are core to our motivation. Other relevant examples include new techniques to address nearby conservation concerns and reductions in CO2 emissions. Such innovations, when realized, are clearly observable.

15. See Laffont and Martimort (2009), Chapter 5.1.2 for details on the properties of the firm's liability rent. From the assumptions made on ψ, it follows that $R' \geq 0$, $R'' \geq 0$ with $R(0) = R'(0) = 0$.

16. Note that $(v'(\pi) - 1)d\pi$ reflects the marginal benefit of relaxing the financial constraint when pledgeable profits increase by $d\pi$. Some of our results depend on the assumption $v'''(\pi) > 0$ (an assumption satisfied in our quadratic example below). This condition amounts to saying that this marginal benefit is convex in π.

but this has only a second-order effect on the expected benefit of innovation. Although the financial constraint is eased, there is not much room to increase this expected benefit and so effort increases at a decreasing rate. This explains the concavity of v.

To illustrate these findings, we provide an example, which, along with the case of frictionless finance, showcases our results and highlights the consequences of relying on costly outside finance.

Costly Finance. Following the analysis in Appendix A, we derive the expression of the firm's endogenous preferences v as

$$v(\pi)=\begin{cases}\pi & \text{for } \pi\in[0,\hat{\pi}),\\ \pi-I+\gamma e^{sb}(\pi)-\psi(e^{sb}(\pi))=R(e^{sb}(\pi)) & \text{for } \pi\in[\hat{\pi},I),\\ \pi-I+R(e^{fb}) & \text{for } \pi\geq I.\end{cases}$$

where $\hat{\pi}=I-e^m R'(e^m)$ and $e^{sb}(\pi)\leq e^{fb}$ stands for the second-best effort over the range of values of π where the innovation is financed by outside financiers.

To illustrate, suppose that $\psi(e)=\frac{\lambda e^2}{2}$ with λ large enough so that optimal effort levels remain interior. Standard computations show that $e^{fb}=\frac{\gamma}{\lambda}$ and $e^m=\frac{e^{fb}}{2}$, while the second-best effort $e^{sb}(\pi)$ is

$$e^{sb}(\pi)=\left(1+\left(1-\frac{I-\pi}{\hat{I}}\right)^{\frac{1}{2}}\right)e^m,\text{ where } \hat{I}=\frac{\gamma^2}{4\lambda}.$$

Tedious but straightforward computations also highlight key properties of the utility function v, namely

$$v'(\pi)=\frac{1}{2}\left(1+\left(1-\frac{I-\pi}{\hat{I}}\right)^{-\frac{1}{2}}\right)>1, v''(\pi)=-\frac{1}{4\hat{I}}\left(1-\frac{I-\pi}{\hat{I}}\right)^{-\frac{3}{2}}<0,$$

$$v'''(\pi)=\frac{3}{8\hat{I}^2}\left(1-\frac{I-\pi}{\hat{I}}\right)^{-\frac{5}{2}}>0.$$

■

Frictionless Finance. Suppose that effort is verifiable and, thus, that there is no moral hazard in the relationship with outside financiers. Outside finance is costless. The innovation is always financed and the firm becomes *de facto* risk neutral:

$$v(\pi)=\pi-I+R(e^{fb}),\quad \forall\pi\geq 0, \tag{3.4}$$

where $R(e^{fb})$ is the expected value of the innovation for e^{fb}. ■

4. COMPLETE INFORMATION BENCHMARK

As a first pass, suppose that the costs θ_1 and θ_2 are both common knowledge, but recall that, at the time of contracting, the cost θ_2 is not yet realized. Following our microfoundations from the previous section, the firm evaluates second-period returns using a utility function v.

The solution to the contracting problem is obvious. First, because costs for the basic service are known and firms are *ex ante* identical with respect to the cost of the add-on, the principal does not have to run an auction to select the most appropriate service provider. She will simply contract with the firm whose cost θ_1 of providing the basic service is known to be the lowest. Second, because transferring risk to a risk-averse firm is costly, the principal should keep all the risk associated with the add-on so as to provide perfect insurance against second-period cost uncertainty.

In this complete information scenario, an optimal profile of intertemporal payments to the winning firm should maximize the expression of intertemporal welfare (2.2) subject to the winning firm's participation constraint:

$$(1-\beta)u_1(\theta_1)+\beta\mathbb{E}_{\theta_2}(v(\pi(\theta_1)+u_2(\theta_1,\theta_2)))\geq 0. \tag{4.5}$$

Proposition 4.1 summarizes the features of an optimal contract under complete information.

Proposition 4.1. *Suppose that $v'(\hat{\pi})>1+\tau>v'(I)$. Under complete information, the following intertemporal profile of payments to the winning firm is optimal:*[17]

$$u_1^{fb}(\theta_1)=-\frac{\beta}{1-\beta}v(\pi^{fb})<0,\pi^{fb}(\theta_1)=\pi^{fb} \text{ and } u_2^{fb}(\theta_1,\theta_2)=0 \tag{4.6}$$

where $\pi^{fb}\in(\hat{\pi},I)$ is defined as

$$v'\left(\pi^{fb}\right)=1+\tau. \tag{4.7}$$

To understand these findings, recall that second-period optimality requires that the public cost of leaving one more euro to the firm equals the firm's marginal utility of income. This condition is pinned down in (4.7). This public cost accounts for the second-period deadweight loss of public funds. The firm's marginal utility of income captures its financial constraints as shown in Section 3 and includes the extra cost that private financiers incur when financing one more euro of the second-period investment. Equation (4.7) defines a level of the firm's second-period profit π^{fb} at which the private cost of outside finance and the public cost of funds are equal. The second element of equation (4.6) shows that the overall second-period profit of the winning firm for the basic service should be precisely equal to that threshold. Even under complete information, second-period payments for the basic service are thus raised accordingly and backloaded. This is in line with casual observations from procurement of public works where certain payments are only executed after the work or project is finalized.[18]

Because, under complete information, the firm's total profits can also be pushed to its reservation payoff, first-period profits are instead negative. The firm make losses on the basic service in the first period and recoup those losses later with extra second-period payments. Note finally that, because of complete information, the principal can exactly

17. The superscript fb stands for *first best* and it indexes optimal variables in the complete information benchmark.

18. We do not consider that firms are in financial distress and can close down as in Board (2007), Calvares et al. (2004) and Burguet et al. (2012). Bankruptcy and moral hazard more generally would be a different reason to backload payments but this justification is not present in our model.

cover the realized costs of providing the add-on and the extra second-period funds reward only the basic service. In the sequel, we will be interested in how asymmetric information might modify this intertemporal structure of the payments.

COSTLY FINANCE (CONTINUED). For future references, we also compute

$$\pi^{fb} = I - \hat{I}\left(1-(1+2\tau)^{-2}\right) < I.$$

As public funds become more costly (τ increasing), more of the investment must be covered by outside financiers which hardens financial constraints. ■

FRICTIONLESS FINANCE (CONTINUED). Suppose that v is given by (3.4). The condition of Proposition 4.1 no longer holds. Using public funds is indeed always more costly than relying on outside finance to pay for the investment. Provided that second-period profits are bound to remain non-negative, we now have a corner solution $\pi^{fb}=0$ where the entire investment is funded by outside finance. ■

These examples illustrate the consequences of relying on costly finance to get an innovation. Even under complete information, payments are backloaded to facilitate funding of costly R&D activities.

5. INCENTIVE FEASIBILITY

We now consider a more realistic scenario which entails asymmetric information on the cost of the basic service. In this section, we maintain the assumption that the cost of the add-on remains verifiable so that the principal directly covers that cost. The case of asymmetric information on the cost of the add-on is dealt with in Section 7.

For a bidding strategy $b(\cdot)$ that is increasing in the cost of the basic service and symmetric across players, the probability that a firm which reports $\hat{\theta}_1$ wins the auction is $(1-F(\hat{\theta}_1))^n$. This allow us to rewrite the requirement of incentive compatibility for a bidder with type θ_1 in terms of its expected payoff as

$$\mathcal{U}(\theta_1) = \max_{\hat{\theta}_1 \in \Theta_1} (1-F(\hat{\theta}_1))^n \left((1-\beta)(b(\hat{\theta}_1)-\theta_1) + \beta v(b(\hat{\theta}_1)-\theta_1+y(\hat{\theta}_1))\right). \tag{5.8}$$

From the Revelation Principle, the above maximum is achieved at θ_1, i.e.,

$$\mathcal{U}(\theta_1) = (1-F(\theta_1))^n ((1-\beta)u_1(\theta_1) + \beta v(\pi(\theta_1))). \tag{5.9}$$

We will consider scenarios where $\pi(\theta_1)$ remains above $\hat{\pi}$ and define $\varphi = v^{-1}$ as the inverse utility function over this range. Actually, $\pi(\theta_1)$ can then be expressed as

$$\pi(\theta_1) = \varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n} - (1-\beta)u_1(\theta_1)}{\beta}\right). \tag{5.10}$$

With this change of variables, any incentive-compatible allocation amounts to a pair $(\mathcal{U}(\theta_1), u_1(\theta_1))$ that stipulates the winning firm's intertemporal payoff and its first-period profit. Equipped with this dual specification of incentive-compatible allocations, we present a lemma which provides necessary but also sufficient conditions for implementability.

Lemma 5.1. NECESSARY CONDITION. *Any incentive-compatible allocation* $(\mathcal{U}(\theta_1),u_1(\theta_1))$ *such that* $\pi(\theta_1)$ *as defined in (5.10) remains above* $\hat{\pi}$*, is such that* $\mathcal{U}(\theta_1)$ *is absolutely continuous in* θ_1 *(and thus almost everywhere differentiable) with at any point of differentiability:*

$$\dot{\mathcal{U}}(\theta_1)=-(1-F(\theta_1))^n\left(1-\beta+\beta v'\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta}\right)\right)\right). \tag{5.11}$$

SUFFICIENT CONDITION. *An allocation is incentive compatible if* $\mathcal{U}(\theta_1)$ *is absolutely continuous, satisfies (5.11) at any point of differentiability and is convex. A sufficient condition for this convexity is* $\pi(\theta_1)$ *non-decreasing.*

To understand the envelope condition (5.11), it is useful to consider the benefits that a firm with per-period cost θ_1 for the basic service gets when pretending to have a marginally higher cost $\hat{\theta}_1=\theta_1+d\theta_1$. Doing so means that this firm can produce the requested basic service at a slightly lower cost even though it reduces the probability of winning to $(1-F(\theta_1+d\theta_1))^n$. The expected cost that is saved with such a manipulation is thus worth $(1-F(\theta_1+d\theta_1))^n\times d\theta_1\approx(1-F(\theta_1))^n d\theta_1$. Since the basic service is run over both periods, this gain is evaluated at the margins $1-\beta$ for the first period and β multiplied by the firm's marginal utility of income in the second period. This marginal utility is itself evaluated at the second-period profit for the basic service which roughly amounts to $u_1(\theta_1+d\theta_1)+y(\theta_1+d\theta_1)-\theta_1\approx\pi(\theta_1)$. Putting these facts together, a firm with cost θ_1 is prevented from mimicking the behavior of a $\theta_1+d\theta_1$ type if it receives an extra information rent worth $\mathcal{U}(\theta_1)-\mathcal{U}(\theta_1+d\theta_1)\approx-\dot{\mathcal{U}}(\theta_1)d\theta_1=(1-F(\theta_1))^n\left(1-\beta+\beta v'(\pi(\theta_1))\right)d\theta_1$. Simplifying yields (5.11).

The right-hand side of (5.11) shows the basic forces at play in the optimal contract. As is familiar in screening environments, the winning firm must get an information rent to reveal its cost for the basic service. By exaggerating its cost, the firm can get extra payments in case it is selected. Yet, this marginal benefit of exaggerating costs must also be weighted with the reduced probability of winning when costs are inflated.

As in Arve and Martimort (2016), here there is also another less familiar *Income Effect* coming from the fact that the firm's second-period marginal utility of income is decreasing. Indeed, a firm with a low cost for the basic service finds it more attractive to exaggerate this cost as its second-period profits are low. To relax incentive compatibility, the principal thus offers a profile of second-period profits that is not only backloaded but also increasing in the cost of the basic service so that any upward cost manipulation becomes less attractive. The firm is then torn between its incentives to exaggerate costs to get a higher rent and the fact that doing so reduces its marginal utility of income. By backloading payments, the principal creates countervailing incentives that reduce the cost of incentives; a phenomenon much in the spirit of findings in Lewis and Sappington (1989) and Khalil and Lawarree (1995).

For a given bidding strategy $b(\theta_1)$ (or alternatively a first-period profit $u_1(\theta_1)=b(\theta_1)-\theta_1$), the rent profile $\mathcal{U}(\theta_1)$ is entirely determined by the differential equation (5.11) for participating types $\theta_1\leq\tilde{\theta}_1$ together with a boundary condition for the cutoff type $\tilde{\theta}_1$:

$$\mathcal{U}(\tilde{\theta}_1)=0. \tag{5.12}$$

Of course, the cutoff type $\tilde{\theta}_1$, i.e., the highest cost that participates, is pinned down by the reserve price that the principal may use to select admissible bidders. This optimal cutoff is investigated in Section 6.2 below. For the time being, notice that integrating (5.11) and taking into account (5.12) yield the following integral representation of the payoff profile:

$$\mathcal{U}(\theta_1) = \int_{\theta_1}^{\tilde{\theta}_1} (1-F(s))^n \left(1-\beta+\beta v'(\pi(s))\right) ds, \quad \forall \theta \leq \tilde{\theta}_1. \tag{5.13}$$

To illustrate the impact of costly outside finance, it is instructive to compare these findings to the example of frictionless finance.

Frictionless Finance (continued). In this scenario, v is given by (3.4). The *Income Effect* then disappears and the information rent takes the familiar right-hand side expression below:

$$\mathcal{U}(\theta_1) = (1-F(\theta_1))^n((1-\beta)u_1(\theta_1)+\beta(\pi(\theta_1)+R(e^{fb})-I)) = \int_{\theta_1}^{\tilde{\theta}_1} (1-F(s))^n ds, \quad \forall \theta \leq \tilde{\theta}_1. \tag{5.14}$$

■

6. OPTIMAL CONTRACT

From the analysis in Section 5, we can conclude that under asymmetric information, two different instruments can a priori be used to extract information rent from the winning firm. First, the principal can play on the intertemporal profile of payments given to that firm. Second, the principal can also decide whether to procure or not the services, i.e., select a cutoff type $\tilde{\theta}_1$ which is the highest admissible cost eligible for a contract.[19] We analyze each instrument in turn. We also characterize bidding behavior in the first-price auction for this long-term contract.

6.1. *Intertemporal Payments Profiles*

Because the firm's incentive compatibility constraint (5.11) depends on how much profit on the basic service is guaranteed for the second period, the principal now chooses an intertemporal distribution of payments so as to relax this constraint. This effect is formally stated in the next proposition.

Proposition 6.1. *Suppose that*

$$v'(I) < 1+\tau+\frac{v''(I^-)}{f(\bar{\theta})}. \tag{6.15}$$

The winning firm's second-period profit $\pi^{ie}(\theta_1)$[20] for the basic service satisfies the following necessary condition for optimality:

$$v'(\pi^{ie}(\theta_1)) = 1+\tau+\frac{F(\theta_1)}{f(\theta_1)} v''(\pi^{ie}(\theta_1)), \quad \forall \theta_1 \leq \tilde{\theta}_1. \tag{6.16}$$

19. This is the equivalent of a reserve price in the direct revelation mechanism.
20. Where the superscript *ie* stands for *Income Effect*.

$\pi^{ie}(\theta_1)$ is non-decreasing in θ_1 when Assumption (2.1) holds and $v'''\geq 0$.

Even if $\pi^{ie}(\theta_1)$ is now greater than the optimal level under complete information, outside finance is still needed when (6.15) holds:

$$\pi^{fb}\leq\pi^{ie}(\theta_1)\leq I, \quad \forall\theta_1\leq\tilde{\theta}_1. \tag{6.17}$$

The profit on the basic service that accrues to the firm in the second period is always beyond the first-best threshold π^{fb}. By raising this profit, the principal eases the firm's access to financial markets. This decreases the firm's marginal utility of income and makes exaggerating the per-period cost for the basic service less attractive. As a by-product, solving the first-period asymmetric information problem also reduces agency concerns on the financial market. The public principal takes a greater share of the second-period investment under asymmetric information; which makes it easier for financiers to enter and decreases the firm's marginal utility of income for the second period.

The second-best optimality condition (6.16) that characterizes second-period profits under asymmetric information can be readily explained. Second-period optimality still requires that the principal's cost of leaving one more euro to the firm equals the firm's marginal utility of income, duly deduced from its financial constraint. The principal's cost, although it still accounts for the second-period deadweight loss of public funds as under complete information (the first term on the right-hand side: $1+\tau$), is now lowered because providing more public funds eases incentive compatibility (the second term, which is negative: $\frac{F(\theta_1)}{f(\theta_1)}v''(\pi^{ie}(\theta_1))<0$). Shifting payments by increasing the second-period profit for high-cost firms makes it less attractive for low-cost firms to mimic the high-cost firms. Although backloading of payments also happens under symmetric information, reducing the information rent calls for shifting even more of the payments to the second period under asymmetric information on θ_1. Of course, there is no need to distort the second-period profit target for the most efficient type $\underline{\theta}_1$; a familiar *no distortion at the top* result.

Costly Finance (continued). Consider the case where θ_1 is uniformly distributed so that $\frac{F(\theta_1)}{f(\theta_1)}=\theta_1-\underline{\theta}_1$. For the purpose of deriving closed-form approximations for payment profiles, we shall now also suppose that τ is small enough. Simultaneously, we will also suppose that asymmetric information on first-period costs is rather mild; a condition that we will write as

$$\tau\geq -v''(I^-)(\overline{\theta}_1-\underline{\theta}_1). \tag{6.18}$$

First-order Taylor expansions for (4.7) and (6.16) then give us

$$\pi^{fb}=I+\frac{\tau}{v''(I^-)}<I \text{ and } \pi^{ie}(\theta_1)=I+\frac{\tau}{v''(I^-)}+\theta_1-\underline{\theta}_1.^{21} \tag{6.19}$$

This example illustrates condition (6.17) in Proposition 6.1. In addition to the backloaded profit π^{fb} that is given under complete information, an extra payment

21. It is immediate to check that the monotonicity condition for second-period profits holds as required by Lemma 5.1.

$\theta_1 - \underline{\theta}_1$ (at most worth $\overline{\theta}_1 - \underline{\theta}_1$) must be given to the firm to facilitate incentive compatibility. ■

FRICTIONLESS FINANCE (CONTINUED). The expression of the information rent (5.14) makes it clear that a whole range of profit pairs $(u_1(\theta_1), \pi(\theta_1))$ leaves the firm's rent unchanged. The fact that there is a positive cost of public funds in the second period implies that the principal optimally chooses $\pi^{ie}(\theta_1) = 0$ when profits must stay non-negative. In other words and in sharp contrast to the case of costly finance, in the second period, the firm is paid solely by means of the net value of the innovation $R(e^{fb}) - I$. ■

6.2. *Optimal Participation*

Let us now turn to the characterization of which firms the principal would like to participate in the tender.[22] Setting a reserve price for the auction implicitly defines a cutoff for the cost of the basic service above which even the least-cost bidder will not be allowed to engage in the long-term project.

Proposition 6.2. *The optimal cutoff $\tilde{\theta}_1^{ie}$, when interior, satisfies the necessary condition*

$$\mathcal{S}(\tilde{\theta}_1^{ie}) + \beta\left(v(\pi(\tilde{\theta}_1^{ie})) - (1+\tau)\pi(\tilde{\theta}_1^{ie})\right) = \frac{F(\tilde{\theta}_1^{ie})}{f(\tilde{\theta}_1^{ie})}(1 - \beta + \beta v'(\pi(\tilde{\theta}_1^{ie}))). \tag{6.20}$$

Condition (6.20) simply means that, for the cutoff $\tilde{\theta}_1^{ie}$, the net benefit of the project including the net present value of the add-on and the net value of the innovation is equal to its *virtual costs*. Of course, the expression of those virtual costs takes into account the *Income Effect*. Because public funds are costly, all costs are also conveniently weighted by the cost of public funds

THE ROLE OF COMPETITION. We observe that the second-period profit $\pi^{ie}(\theta_1)$ as defined in (6.16) and the participation cutoff $\tilde{\theta}_1^{ie}$ defined in (6.20) are both independent of the number of bidders participating in the auction. The sole role of competition is thus to reduce the probability that any single firm ends up being the lowest-cost one and is awarded the project. Competition extracts the firms' information rent by making it less likely for an individual firm to become the provider of the service, but it does not affect other features of the contractual relationship. The intuition is straightforward. Once selected, the lowest-cost bidder remains in a bilateral relationship with the principal. Given this selection, the screening devices available to further extract rent beyond the selection rule, i.e., the intertemporal structure of profits and the participation cutoff, remain the same and are distorted in the same way with or without competition. This result thus echoes other standard results in the auction literature. As far as unit auctions are concerned, Myerson (1981) and Riley and Samuelson (1981) have for instance shown that the optimal reserve price is independent of the number of competing bidders. In

22. In sharp contrast to Arve and Martimort (2016), the principal can no longer adjust quantities for screening purposes since the sizes of the basic service and the add-ons are both fixed to one unit. However, in this paper a new feature is that the principal may exclude firms from the tender.

the case of auctions for incentive contracts, McAfee and McMillan (1986), Riordan and Sappington (1987) and Laffont and Tirole (1987) have also demonstrated that competition reduces payments to bidders but does not modify how much the winning firm would produce conditionally on having been selected.

Frictionless Finance (continued). To understand Proposition 6.2, it is useful to consider the case of frictionless access to the financial market which always leads to the implementation of the innovation of value $R(e^{fb})-I$. The following corollary characterizes participation in that case.

Corollary 6.3. *When finance is frictionless, the optimal cutoff $\tilde{\theta}_1^{rn}$, when interior, satisfies the necessary condition:*

$$\mathcal{S}(\tilde{\theta}_1^{rn})+\beta(1+\tau)(R(e^{fb})-I)=\frac{F(\tilde{\theta}_1^{rn})}{f(\tilde{\theta}_1^{rn})}. \tag{6.21}$$

Provided this condition is also sufficient, participation is facilitated when finance is frictionless:

$$\tilde{\theta}_1^{ie}\leq\tilde{\theta}_1^{rn}. \tag{6.22}$$

Although backloading of payments relaxes incentives related to θ_1, it does not fully eliminate the incentive problem and its costs. This corollary indicates that the remaining cost of outside finance with moral hazard frictions ends up being borne by society and the principal therefore restricts participation accordingly. ■

6.3. *Bidding Strategies and the Reserve Price*

We now turn to the characterization of bidding strategies in the first-price auction. We look for a symmetric increasing equilibrium bidding strategy $b^{ie}(\theta_1)$ for our first-price auction setting. As explained in Section 2, this auction is made somewhat complex by the fact that the second-period profit is also determined by the first-period bid.[23] Moreover, the auction stipulates a reserve price equal to $b^{ie}(\tilde{\theta}_1^{ie})$. The next proposition characterizes the equilibrium bidding strategy and provides a sufficient condition for monotonicity.

Proposition 6.4. *For a given cutoff $\tilde{\theta}_1$, the equilibrium bidding strategy in the first-price auction satisfies*

$$(1-\beta)(b^{ie}(\theta_1)-\theta_1)+\beta v(\pi^{ie}(\theta_1))=\frac{1}{(1-F(\theta_1))^n}\int_{\theta_1}^{\tilde{\theta}_1^{ie}}(1-F(s))^n\left(1-\beta+\beta v'\left(\pi^{ie}(s)\right)\right)ds. \tag{6.23}$$

The bidding strategy $b^{ie}(\cdot)$ is non-decreasing provided that β is small enough.

To understand the nature of the bidding strategy and some key comparative statics, our two leading examples are again useful.

23. The second-period payment is equal to the bid b plus an additional payment $y^{ie}(b^{i-1}(b))$, where the expression of $y^{ie}(\theta_1)=\pi^{ie}(\theta_1)-u_1^{ie}(\theta_1)$ directly follows from Proposition 6.1.

Frictionless Finance (continued). In this scenario, the bidding strategy is given by

$$(1-\beta)(b^{ie}(\theta_1)-\theta_1)+\beta(R(e^{fb})-I)=\frac{1}{(1-F(\theta_1))^n}\int_{\theta_1}^{\tilde{\theta}_1^{ie}}(1-F(s))^n ds. \tag{6.24}$$

The right-hand side is nothing else than the bid markup $b^0(\theta_1)-\theta_1$ that would be chosen if only the basic service was up for tender (which amounts to setting $\beta=0$ in our analysis). It is the familiar mark-up for a unit auction. With the add-on, the winning firm is paid a fixed amount for the second period independently of its type. This fixed amount is the value of the innovation $R(e^{fb})-I$. All strategic considerations in bidding thus come from the first period. In particular, when the second period matters more (β increasing), bidding in the first period becomes less aggressive, with greater markups $b^{ie}(\theta_1)-\theta_1$. ■

Costly Finance (continued). Consider the case where θ_1 is uniformly distributed and suppose again that τ and $\overline{\theta}_1-\underline{\theta}_1$ are small enough so as to satisfy (6.18). Because asymmetric information is small in magnitude, there is full participation and $\tilde{\theta}^{ie}=\overline{\theta}_1$. A first-order Taylor approximation gives us

$$v(\pi^{ie}(\theta_1))=v(I)+v'(I)(\pi^{ie}(\theta_1)-I)=R(e^{fb})-I+\pi^{ie}(\theta_1).$$

Using these two facts, the bidding strategy can then be approximated by

$$(1-\beta)(b^{ie}(\theta_1)-\theta_1)+\beta\left(R(e^{fb})+\frac{\tau}{v''(I^-)}+\theta_1-\underline{\theta}_1\right)=\frac{1}{n+1}(\overline{\theta}_1-\theta_1). \tag{6.25}$$

As public funds become more costly (τ increasing) and outside finance more difficult, the firm becomes endogenously more risk averse and, accordingly, bids more aggressively.[24] ■

Reserve Price. To implement the optimal cutoff for participation $\tilde{\theta}_1^{ie}$, the principal imposes a reserve price on the auction which is given by

$$b^{ie}(\tilde{\theta}_1^{ie})=\tilde{\theta}_1^{ie}-\frac{\beta}{1-\beta}v(\pi^{ie}(\tilde{\theta}_1^{ie})). \tag{6.26}$$

It is worth noticing that this price is below the cost $\tilde{\theta}_1^{ie}$. This means that the principal may induce a loss in the first period on some participating agents. This loss is then compensated by the value of the innovation that the winning firm can get from the second-period add-on.

24. This result is in line with results in the auction literature and especially Holt (1980), Hu et al. (2010), Krishna (2002), Maskin and Riley (1984) Matthews (1983), Matthews (1987) and Milgrom and Weber (1982). These authors have shown that in a standard first-price auction for a single item, risk-averse bidders bid more aggressively than if they were risk neutral. This is because, for a risk-averse bidder compared with a risk-neutral bidder, the risk of losing the auction from a small increase in the bid has a larger effect on expected utility than the loss of profits from a slightly lower bid. A risk-averse bidder would thus be willing to lower his bid more than under risk neutrality to reduce the risk of loosing the auction. The key difference with this literature is that, in our context, risk aversion is endogenous.

7. ASYMMETRIC INFORMATION ON THE COST OF THE ADD-ON

We now extend the analysis in Section 6 to a scenario with asymmetric information on the cost of the add-on. Under these circumstances, the firm, endowed with the endogenous preferences v, must also absorb some endogenous risk induced by incentive compatibility constraints related to the cost of the add-on.

7.1. *Endogenous Utility Function*

To characterize how the firm and the principal should react to the additional risk induced by incentive compatibility constraints related to the cost of the add-on, we follow an approach borrowed from Arve and Martimort (2016). Consider first a random variable $\tilde{\pi}$ that takes the values $\pi+(1-\nu)\Delta$ and $\pi-\nu\Delta$ with respective probabilities ν and $1-\nu$. We then define $w(\pi,\Delta)$, a utility function over expected return π and risk level $\Delta\geq 0$ as $w(\pi,\Delta)\equiv\mathbb{E}_{\tilde{\pi}}(v(\tilde{\pi}))$. The function w clearly inherits some important properties from the underlying utility function v as it is also increasing and concave in π. Because v is strictly concave over the range $[\hat{\pi},I)$ where raising outside finance is costly, w is also decreasing in Δ as long as the random variable $\tilde{\pi}$ remains within this support. This monotonicity in Δ captures the fact that more background risk, represented by a higher value of Δ, reduces the firm's expected payoff. At the other extreme, the case $\Delta=0$ corresponds to the certainty case with $w(\pi,\Delta)\equiv v(\pi)$.

The next corollary informs us on how more risk (a higher value of Δ) modifies the firm's marginal utility of income. It provides a key step to understand how the addition of asymmetric information on the cost of the add-on impacts first-period incentives.

Corollary 7.1. *When $v'''\geq 0$ and $\tilde{\pi}\in[\hat{\pi},I)$, w also satisfies $w_{\pi\Delta}(\pi,\Delta)\geq 0$.*

Costly Finance (continued). The payoff function v satisfies $v'''\geq 0$ on $[\hat{\pi},I)$ and Corollary 7.1 applies. Observe that, when Δ is small enough, the random variable $\tilde{\pi}$ also has support on $[\hat{\pi},I)$ if π belongs to that interval. The following approximation for the expected payoff $w(\pi,\Delta)$ then holds up to terms of order more than two:

$$w(\pi,\Delta)=v(\pi)-\frac{\nu_1(1-\nu)\Delta^2}{8\hat{I}}\left(1-\frac{I-\pi}{\hat{I}}\right)^{-\frac{3}{2}} \text{ and } w_{\pi\Delta}(\pi,\Delta)=\frac{3\nu_1(1-\nu)\Delta}{8\hat{I}^2}\left(1-\frac{I-\pi}{\hat{I}}\right)^{-\frac{5}{2}}>0. \tag{7.27}$$

From this, we also get that $w_{\pi\pi}(\pi,\Delta)\leq v''(\pi)$. In other words, the addition of a background risk (that in the sequel will be due to second-period asymmetric information) makes the firm less averse to risk. ■

7.2. *The Risk Effect*

Second-period incentive compatibility. Following Baron and Besanko (1984), Battaglini (2005) and Pavan et al. (2014), the requirement of incentive compatibility can be applied recursively. For any report $\hat{\theta}_1$ of the per-period cost of the basic service that a winning firm may have reported in the first period, the requirement of incentive compatibility implies that the second-period report of the cost of the add-on, which is truthful from the Revelation Principle, should maximize the firm's continuation payoff

$p(\hat{\theta}_1,\hat{\theta}_2)-\theta_2$.[25] Because the add-on is a fixed-size project that is always valuable, no quantity screening can be used to help rent extraction in the second period. The winning firm will thus be paid a fixed price $p(\hat{\theta}_1,\hat{\theta}_2)=p(\hat{\theta}_1)$ for the provision of the add-on that may only depend on its announcement of the cost of the basic service. The firm's second-period profit from the add-on is thus

$$u_2(\hat{\theta}_1,\theta_2)=p(\hat{\theta}_1)-\theta_2. \tag{7.28}$$

Because any non-zero expected profit from this add-on could, by a simple redefinition of payments, be incorporated into the second-period premium for the basic service, $y(\hat{\theta}_1)$, there is no loss of generality in assuming that the firm makes zero expected profit on the standard version of the add-on. This means that the second-period price for the add-on covers the expected cost of its standard version and is thus independent of the first-period announcement of the cost of the basic service:

$$p(\hat{\theta}_1)=\mathbb{E}_{\theta_2}(\theta_2),\ \forall\hat{\theta}_1\in\Theta_1. \tag{7.29}$$

Second-period profits from the add-on can thus be expressed as a random variable with zero mean:

$$u_2(\underline{\theta}_2)=(1-\nu)\Delta \text{ and } u_2(\overline{\theta}_2)=-\nu\Delta. \tag{7.30}$$

Second-period incentive compatibility thus imposes an endogenous risk Δ on the firm's second-period returns.

First-period incentive compatibility. Using our definition of w, we may now generalize our integral representation of the firm's intertemporal payoff in (5.13)[26] to take into account this endogenous risk and get

$$\mathcal{U}(\theta_1)=\int_{\theta_1}^{\tilde{\theta}_1}(1-F(s))^n(1-\beta+\beta w_\pi(\pi(s),\Delta))ds, \quad \forall\theta\le\tilde{\theta}_1. \tag{7.31}$$

Optimal contract. Proposition 7.2 summarizes our findings for the optimal contract with asymmetric information on θ_2.

Proposition 7.2. *Suppose that θ_2 is private information and that*

$$w_\pi(I,\Delta)<1+\tau+\frac{w_{\pi\pi}(I,\Delta)}{f(\overline{\theta})}. \tag{7.32}$$

The winning firm's second-period expected profit $\pi^{sb}(\theta_1,\Delta)$ for the basic service satisfies the following necessary condition for optimality:

$$w_\pi(\pi^{sb}(\theta_1,\Delta),\Delta)=1+\tau+\frac{F(\theta_1)}{f(\theta_1)}w_{\pi\pi}(\pi^{sb}(\theta_1,\Delta),\Delta), \quad \forall\theta_1\le\tilde{\theta}_1. \tag{7.33}$$

$\pi^{sb}(\theta_1,\Delta)$ is non-decreasing in θ_1 when Assumption (2.1) holds and $v'''\ge 0$. It is also greater than in the case where θ_2 is verifiable when $v'''\ge 0$ and $v''''\le 0$:

$$\pi^{ie}(\theta_1)\le\pi^{sb}(\theta_1,\Delta), \quad \forall\theta_1\le\tilde{\theta}_1. \tag{7.34}$$

25. We omit the index i_0 of the winning firm for simplicity.
26. See proof in the Appendix ("Proof of Lemma 5.1 and the general case in Section 7").

Moreover, outside finance is still needed when (7.32) holds:

$$\pi^{sb}(\theta_1,\Delta)\leq I, \quad \forall \theta_1 \leq \tilde{\theta}_1. \tag{7.35}$$

To further illustrate this *Risk Effect* and stress the role of second-period asymmetric information on the costs of the add-on, it is again useful to come back to our leading examples.

Frictionless Finance (continued). Because v is linear with frictionless finance, all second-period risk disappears when taking expectations. This means that asymmetric information on second-period costs has no impact on first-period incentives and all the results of Section 6 obtained when outside finance is frictionless carry over. ■

Costly Finance (continued). Assume that θ_1 is uniformly distributed. On top of assuming that τ and $\overline{\theta}_1-\underline{\theta}_1$ are both small enough, let us also consider a scenario with a small degree of asymmetric information on second-period costs of the add-on, i.e., Δ^2 is small enough. More specifically, we shall impose τ small together with a strengthening of Condition (6.18), namely

$$\tau \geq -v''(I^-)(\overline{\theta}_1-\underline{\theta}_1)+\frac{5\nu_1(1-\nu)\Delta^2}{16\hat{I}^2}. \tag{7.36}$$

Using the approximation (7.27) in (7.33), we then get the following closed-form expression of second-period profits up to terms of higher order magnitude:

$$\pi^{sb}(\theta_1,\Delta)=\pi^{ie}(\theta_1)-\frac{5\nu_1(1-\nu)\Delta^2}{16\hat{I}^2v''(I^-)}\in(\pi^{ie}(\theta_1),I), \quad \forall\theta_1\in\Theta_1,$$

where the upper bound follows from (7.36). This expression shows that to cope with the endogenous risk induced by asymmetric information on the cost of the add-on, profits on the basic service have to include an extra risk premium $-\frac{5\nu_1(1-\nu)\Delta^2}{16\hat{I}^2v''(I^-)}$. This introduces another element that backloads payments in addition to the effects identified in Propositions 4.1 and 6.1. This premium increases with the magnitude of the asymmetric information problem on the add-on. It also increases when agency frictions with financiers (captured by the quantity $-v''(I^-)$) are of lower magnitude. ■

Proposition 7.3. *Suppose that the firm has private information on θ_2. The optimal cutoff $\tilde{\theta}_1^{sb}$, when interior, satisfies the necessary condition*

$$\mathcal{S}(\tilde{\theta}_1^{sb})+\beta\Big(w(\pi^{sb}(\tilde{\theta}_1^{sb},\Delta),\Delta)-(1+\tau)\pi^{sb}(\tilde{\theta}_1^{sb},\Delta)\Big)=\frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(1-\beta+\beta w_\pi(\pi^{sb}(\tilde{\theta}_1^{sb},\Delta),\Delta)). \tag{7.37}$$

Provided that this necessary condition is also sufficient, and that Corollary 7.1 holds, participation is always reduced in comparison with the case where θ_2 is verifiable:

$$\tilde{\theta}_1^{sb}<\tilde{\theta}_1^{ie}. \tag{7.38}$$

The second-period endogenous risk Δ has to be borne by the firm when it has private information on the add-on. This decreases utility levels. To induce participation, the principal must pay the firm an extra risk premium which makes inducing participation less attractive, thus lowering the threshold cost of participation compared to the case with $\Delta = 0$.

Remark. It is straightforward to show that all results in this section also hold in an alternative model where there is no add-on, but there is an additive shock to costs for the basic service in the second period and the principal wants to offer an additional payment that compensates for such a shock. ■

8. CONCLUDING REMARKS

In this paper, we consider a procurement auction for the provision of a long-term service to which an add-on, whose costs are ex ante uncertain, must later be appended. Based on the possibility for the firm to enhance the value of the add-on through its R&D activities, we show that costly financing based on agency frictions of such R&D investments makes the firm behave as if it was risk averse in the second period.

To facilitate outside funding of the R&D investment, it is optimal for the procurement agency to backload payments even though there is a cost of public funds associated with doing so. This is in sharp contrast to the case of frictionless finance, in which case all payments are shifted to the initial period, thus avoiding additional costs of public funds. This implies that the public procurement agency (and ultimately society) bears the cost of easing the incentive problem and the financial constraint of the firm. To reduce this cost, participation in the auction is restricted for high-cost firms. These results add to and complement insights in Arve and Martimort (2016) who do not consider effects on the extensive margin (nor do they provide a micro-foundation for second-period risk aversion).

In terms of bidding, risk aversion makes firms bid more aggressively and, in fact, for high enough cost realizations, initial bids will be below cost. Upon winning, this is compensated both by the agency rent from the R&D investment as well as backloaded payments in the optimal contract. This suggests that small extra shocks to costs may easily lead to bankruptcy if firms had to be protected by limited liability in the first period as well.

To obtain clear-cut results, we have deliberately kept the modelling environment as simple as possible. For instance, we do not allow for asymmetric cost distributions (see Myerson (1981) for a general result and Menicucci (2003) for a specific result with risk-averse bidders), asymmetric risk aversion (as in for instance Maréchal and Morand (2011) and Menicucci (2003)) or technological linkage (in terms of correlation across costs for the basic service and the add-on as in the vast literature on dynamic mechanism design[27]) as we believe this would reduce the readability of our results regarding how second-period uncertainty and risk aversion affects bidding, contracting and competition.

27. This literature stresses the value of history in long-term relationships, especially when types are serially correlated (Battaglini (2005), Zhang (2009), Battaglini and Lamba (2019), Esö and Szentes (2017), Kapicka (2013), Garrett and Pavan (2015)), and when current projects affect future technological frontier (Lewis and Yildirim (2002), Gärtner (2010), Auray et al. (2011)).

Our model could be extended along several dimensions. First, the mere possibility of having to provide an add-on could be uncertain at the time of drafting a long-term contract for the basic service.[28] This additional layer of risk certainly increases the risk premium that must be paid by the principal to induce firms to participate in the first place. We expect that under those circumstances, both the *Income* and the *Risk Effects* (when applied to another indirect utility function that would account for this extra risk) will be exacerbated. As a result, backloading payments for the basic service might be found even more attractive.

Second, and following in that respect existing practices,[29] we have assumed that the add-on was absolutely needed and of fixed size. The possibility of scaling down this additional project, or even to completely shut it down, is of course attractive for incentive compatibility reasons in the second period. Scaling down the project if reported costs for the add-on are high reduces the risk borne by the firm at this date and eases second-period incentive compatibility. Both the *Income* and the *Risk Effects* are of a lower magnitude. In other words, scaling down the project goes in the direction of making backloading payments less attractive.

In turn, the possibility of playing on the size of the add-on also has consequences on first-period incentives. Echoing the intuition from optimal participation (reserve price) which suggests that long-term projects with add-ons surrounded with much uncertainty should be under a hard budget constraint, the principal might want to commit to an undersized add-on to improve first-period incentives for revealing costs on the basic service and increase participation. Yet, once the second period comes along and those costs are known, this reason for distorting the size of the add-on has disappeared. Still, the principal may want to scale down this add-on to improve second-period incentive compatibility but these motives yield distortions of a lower magnitude. In other words, the long-term contract that was drafted *ex ante* should be renegotiated. The possibility of such renegotiations will of course be anticipated and may perturb first-period incentive compatibility. It should be noted that, although similar in spirit to the phenomenon highlighted in the earlier literature on renegotiation in adverse selection environments (Dewatripont (1989), Hart and Tirole (1988), Laffont and Tirole (1990)), the effect that we uncover here is novel. While the earlier literature focuses on persistent types, our setting has costs which are independently distributed over time and tasks, and the interplay between second- and first-period incentives only comes from the concavity of second-period payoff. The characterization of optimal renegotiation-proof arrangements in such an environment awaits further analysis.

Lastly, we have assumed that the add-on was necessarily performed by the firm winning the first-price auction for the basic service. The implicit assumption, much in line with existing practices, was that running a second-period auction for the provision of the add-on was too costly, maybe because of administrative or other transaction costs. The possibility of organizing such an auction would have various consequences. First, the probability of being the provider would *de facto* decrease for the winning firm. This effect is very similar to a downscaling of the add-on and would have the same consequences on first- and second-period incentives and on the scope for renegotiation. Second, running

28. Another possibility is to combine insights and elements from our dynamic framework with a multidimensional environment à la Che (1993) or Asker and Cantillon (2008, 2010).

29. See Riksrevisionen (2021).

a second-period auction would allow the principal to improve second-period efficiency by selecting a more efficient provider for the add-on; an issue that was touched upon by the literature on repeated franchise contracts in a different context (Laffont and Tirole (1988), Riordan and Sappington (1989)).

We hope to investigate some of these issues in future research.

Appendix

Appendix A

This appendix provides a full treatment of the endogenization of the firm's payoff function v. Proposition 8.1 below presents the shape of this function under very general conditions.

Incentive feasibility. We first start by describing the set of incentive-feasible financial contracts. To accept outside financing for the investment I, the firm's expected payoff from doing so needs to be greater than its stable return π:

$$\pi+e(\gamma-\overline{T})-(1-e)\underline{T}-\psi(e)\geq\pi. \tag{8.39}$$

Effort being non-verifiable, it is chosen by the firm so as to maximize its own payoff. For any interior solution, the firm's moral hazard constraint can thus be expressed by means of the first-order approach:

$$\gamma+\underline{T}-\overline{T}=\psi'(e). \tag{8.40}$$

Finally, the firm can use the return π as collateral to secure its loan; which leads to the following limited liability constraint:

$$\pi-\underline{T}\geq 0. \tag{8.41}$$

The Case of a Monopolistic Lender. It is useful to first present a benchmark scenario where the innovation is financed by a monopolistic lender. In this scenario, we provide conditions that ensure that the effort e^m defined in (3.3) is implemented. Formally, the monopolistic lender's problem writes as

$$\max_{e\in[0,1],\underline{T},\overline{T}} e\overline{T}+(1-e)\underline{T}-I \text{ subject to (8.39), (8.40), and (8.41).}$$

Suppose first that the firm's participation constraint (8.39) is slack at the optimum. At the solution to this relaxed problem both (8.40) and (8.41) are thus binding. Then, the monopolistic lender's objective boils down to maximizing

$$\max_{e\in[0,1]} \pi+e\gamma-\psi(e)-R(e)-I.$$

The optimal solution is e^m as defined in (3.3).

Finally, observe that, for this solution, the firm's participation constraint (8.39) is slack if

$$\pi+e^m(\gamma-\overline{T})-(1-e^m)\underline{T}-\psi(e^m)=R(e^m)>\pi. \tag{8.42}$$

This condition requires that the firm's stable return π is not too large so that the financial contract remains attractive to the firm.

Let us now define the threshold $\hat{\pi}$ so that the value of the monopolistic lender's maximization problem is zero, thus leaving this lender, whose financial contract would be accepted by the firm, indifferent between financing or not the innovation. The maximum amount, $I-\hat{\pi}$, that this monopolistic lender would be ready to finance would just equal the overall surplus at that effort level minus the liability rent $R(e^m)$ that would be left to the firm in that scenario. Formally, we have

$$I-\hat{\pi}=e^m\gamma-\psi(e^m)-R(e^m)\Leftrightarrow\hat{\pi}=I-e^mR'(e^m)<I. \tag{8.43}$$

From now on, we shall assume that

$$\hat{\pi}\geq 0. \tag{8.44}$$

This condition simply means that if the stable return π, although positive, remains below $\hat{\pi}$, even a monopolistic lender inducing effort e^m would refuse to provide funding. Taking into account (8.42), the monopolistic lender implements e^m when

$$\pi\in[\hat{\pi},R(e^m)]. \tag{8.45}$$

Competitive Financiers. The condition for financiers to break even, i.e., expected loan reimbursements cover the investment outlay, writes as:

$$e\overline{T}+(1-e)\underline{T}-I=0. \tag{8.46}$$

Much in the spirit of Holmström and Tirole (1997), but allowing for effort being a continuous variable, a firm endowed with stable return π lets outside financiers compete in contracts to maximize its expected profit subject to the break-even, the incentive and the limited liability constraints above. Formally, this problem writes as

$$\max_{e\in[0,1],\underline{T},\overline{T}} \pi+e(\gamma-\overline{T})-(1-e)\underline{T}-\psi(e) \text{ subject to (8.40), (8.41) and (8.46).} \tag{8.47}$$

Undertaking the investment is attractive for the firm whenever the solution to this problem also satisfies (8.39). Let $v(\pi)$ be the value of the firm's problem defined in (8.47).

Proposition 8.1. *When a firm endowed with stable return π lets outside financiers compete in contracts to maximize its expected profit subject to the break-even, the incentive and the limited liability constraints as described in (8.47), then the firm's payoff function $v(\pi)$ and the corresponding effort level $e^{sb}(\pi)$ it chooses are defined as follows:*

1. *For $\pi\in[0,\hat{\pi})$, the innovation cannot be financed:*

$$v(\pi)=\pi \text{ and } e^{sb}(\pi)=0. \tag{8.48}$$

2. *For $\pi\in[\hat{\pi},I)$, the innovation can be financed but the firm's effort is suboptimal:*

$$v(\pi)=R(e^{sb}(\pi))=\pi-I+\gamma e^{sb}(\pi)-\psi(e^{sb}(\pi)). \tag{8.49}$$

In particular, $e^{sb}(\pi)$ is non-decreasing in π, $e^m\leq e^{sb}(\pi)<e^{fb}$ and, provided condition (3.3) holds, $v(\pi)\geq\pi$.

3. For $\pi \geq I$, the innovation can be self-financed by the firm and effort is set at its first-best level: $v(\pi) = \pi - I + R(e^{fb})$.

Proof. Proof of Proposition 8.1 First, expressing $\overline{T}$ and $\underline{T}$ from (8.46) and (8.40), we may rewrite (8.41) as a single incentive-feasibility condition, namely

$$e\gamma - \psi(e) - R(e) \geq I - \pi. \tag{8.50}$$

It is straightforward to check that, reciprocally, if (8.50) holds, there exists $\overline{T}$ and $\underline{T}$ such that (8.46), (8.40) and (8.41) all hold.

Second, inserting (8.46) into the maximand (8.47) yields an expression of the firm's objective as:

$$\pi - I + e\gamma - \psi(e). \tag{8.51}$$

The firm's problem, when expressed in terms of effort only, boils down to maximizing (8.51) subject to (8.50).

From the assumptions made on ψ, the left-hand side of (8.50), which we denote $\Gamma(e)$, is a strictly concave function of e. Γ is maximized at e^m and $\Gamma(e^m) = e^m R'(e^m)$. Item 1 immediately follows. The constrained set defined by (8.50) is empty when $\pi \in [0, \hat{\pi})$ where $\hat{\pi} = I - e^m R'(e^m)$, i.e., the innovation cannot be financed by competitive financiers on this range. Therefore, (8.48) follows.

A contrario, notice that e^{fb} maximizes (8.51). The first-best effort level is thus feasible when (8.50) holds for e^{fb}, which requires $\pi \geq I$ (Item 3).

For intermediate levels of base profit (Item 2), (8.50) is binding which determines the value of $e^{sb}(\pi)$ as in (8.49). When $\pi \in [\hat{\pi}, I)$, there are two solutions to the corresponding equation $\Gamma(e) = I - \pi$. The solution that maximizes the firm's expected payoff is the highest one, which is closer to the first-best e^{fb}, and it is on the decreasing branch of $\Gamma(e)$. The fact that $e^{sb}(\pi)$ is non-decreasing in π on this interval immediately follows. Because $I \geq \pi$ on this interval and $\Gamma(e^{fb}) = 0$, the greatest solution to $\Gamma(e) = I - \pi$ is below the first-best e^{fb}.

Finally, it must be that (8.39) holds, i.e., $v(\pi) = R(e^{sb}(\pi)) \geq \pi$ for $\pi \in [\hat{\pi}, I)$ so that the firm prefers to accept the financial contract and undertake the innovation rather than enjoying its stable returns only. This condition writes as

$$\gamma e^{sb}(\pi) - \psi(e^{sb}(\pi)) \geq I \quad \forall \pi \in [\hat{\pi}, I).$$

Because $e^{sb}(\pi)$ is non-decreasing in π and below the first-best e^{fb}, this condition holds if it already holds for $\hat{\pi}$ and there $e^{sb}(\hat{\pi}) = e^m$ so that the corresponding condition writes as (3.3). In particular, v has an upward discontinuity at $\hat{\pi}$ when (3.3) holds as a strict inequality. ‖

Proposition 8.1 has the following implications.

Corollary 8.2. *The payoff function v is non-decreasing, linear on $[0, \hat{\pi}) \cup [I, +\infty)$, strictly concave on $(\hat{\pi}, I)$ with a derivative $v'(\pi) > 1$ on that interval, and an upward discontinuity at $\hat{\pi}$ with $v'(\hat{\pi}^+) = +\infty$ when (3.3) holds as a strict inequality.*

Proof. Proof of Corollary 8.2 That the payoff function $v(\pi)$ is non-decreasing and linear on $\pi \in [0,\hat{\pi}) \cup [I,+\infty)$ immediately follows from Proposition 8.1. Because $\hat{\pi} = I - e^m R'(e^m)$, (3.3), when strict, implies

$$v(\hat{\pi}^+) = R(e^m) = \hat{\pi} - I + e^m \gamma - \psi(e^m) > \hat{\pi} = v(\hat{\pi}^-).$$

To prove that v is strictly concave on $(\hat{\pi}, I)$, consider two profit levels π_1 and π_2 in that interval with the associated second-best effort levels $e^{sb}(\pi_1)$ and $e^{sb}(\pi_2)$. Because (8.50) is binding at those effort levels, we have

$$\pi_i - I + e^{sb}(\pi_i)\gamma - \psi(e^{sb}(\pi_i)) - R(e^{sb}(\pi_i)) = 0, \quad i = 1,2.$$

Fix $\mu \in [0,1]$ and consider $\bar{\pi} = \mu\pi_1 + (1-\mu)\pi_2 \in \left[I - e^m R'(e^m), I\right]$ and $\bar{e} = \mu e^{sb}(\pi_1) + (1-\mu)e^{sb}(\pi_2)$. Because Γ (as defined in the Proof of Proposition 8.1) is strictly concave and the assumptions on ψ, we have

$$\bar{\pi} - I + \bar{e}\gamma - \psi(\bar{e}) - R(\bar{e}) >$$

$$\mu(\pi_1 - I + e^{sb}(\pi_1)\gamma - \psi(e^{sb}(\pi_1)) - R(e^{sb}(\pi_1))) + (1-\mu)(\pi_1 - I + e^{sb}(\pi_2)\gamma - \psi(e^{sb}(\pi_2)) - R(e^{sb}(\pi_2))) = 0$$

and $\bar{e}$ is thus feasible at profit $\bar{\pi}$.

Because the maximand (8.51) is also strictly concave, we have

$$\bar{\pi} - I + \bar{e}\gamma - \psi(\bar{e}) >$$

$$\mu(\pi_1 - I + e^{sb}(\pi_1)\gamma - \psi(e^{sb}(\pi_1))) + (1-\mu)(\pi_1 - I + e^{sb}(\pi_2)\gamma - \psi(e^{sb}(\pi_2))) = \mu v(\pi_1) + (1-\mu)v(\pi_2).$$

Hence,

$$v(\bar{\pi}) \geq \bar{\pi} - I + \bar{e}\gamma - \psi(\bar{e}) > \mu v(\pi_1) + (1-\mu)v(\pi_2),$$

which concludes the proof of strict concavity on $(\hat{\pi}, I)$.

Differentiating the definition of $v(\pi)$ found in (8.49) yields

$$v'(\pi) = 1 + e^{sb'}(\pi)(\gamma - \psi'(e^{sb}(\pi))) = 1 + \frac{e^{sb'}(\pi)(I-\pi)}{e^{sb}(\pi)} \tag{8.52}$$

where the second equality follows from (8.50) being binding on $[\hat{\pi}, I)$ and where

$$e^{sb'}(\pi) = -\frac{1}{\gamma - \psi'(e^{sb}(\pi)) - R'(e^{sb}(\pi))} > 0 \tag{8.53}$$

since $e^{sb}(\pi) \geq e^m$.

Finally, notice that $v'(\hat{\pi}^+) = e^{sb'}(\hat{\pi}^+) = +\infty$. ‖

As explained in the main text, Corollary 8.2 implies that more base profit facilitates access to outside finance by relaxing financial constraints. Yet, it does so at a lower marginal rate and this explains the concavity of the firm's payoff function at higher profit levels and the decreasing marginal utility exhibited by the endogenous preferences v. This result also implies that access to outside finance requires a minimum profit level.[30]

30. Those preferences may also exhibit an upward discontinuity at the profit level where outside financiers are ready to jump in when (3.3) is strict. Because v may exhibit such an upward discontinuity

Appendix B

Proof. Proof of Proposition 4.1 We denote by ζ the non-negative Lagrange multiplier for (4.5) and form the corresponding Lagrangean of the optimization problem in the text as

$$\mathcal{L}(u_1(\theta_1),\pi(\theta_1),u_2(\theta_1,\theta_2),\zeta)=\mathcal{S}(\theta_1)-(1-\beta)u_1(\theta_1)-\beta(1+\tau)\left(\pi(\theta_1)+\mathbb{E}_{\theta_2}(u_2(\theta_1,\theta_2))\right)$$

$$+\zeta\left((1-\beta)u_1(\theta_1)+\beta\mathbb{E}_{\theta_2}(v(\pi(\theta_1)+u_2(\theta_1,\theta_2)))\right).$$

The necessary (and sufficient, thanks to the concavity of this Lagrangean) conditions for optimality with respect to $u_1(\theta_1)$, $\pi(\theta_1)$ and $u_2(\theta_1,\theta_2)$ are respectively given by

$$-(1-\beta)+\zeta(1-\beta)=0, \tag{8.54}$$

$$-\beta(1+\tau)+\zeta\beta\mathbb{E}_{\theta_2}\left(v'(\pi(\theta_1)+u_2(\theta_1,\theta_2))\right)=0, \tag{8.55}$$

$$-\beta(1+\tau)+\zeta\beta v'(\pi(\theta_1)+u_2(\theta_1,\theta_2))=0. \tag{8.56}$$

Simplifying (8.54) yields

$$\zeta=1. \tag{8.57}$$

Condition (8.56) holds for both realizations of θ_2. It thus follows that $u_2^{fb}(\theta_1,\theta_2)$ is independent of θ_2 and, moreover, we have

$$\pi^{fb}(\theta_1)+u_2^{fb}(\theta_1,\theta_2)=\pi^{fb},\quad \forall\theta_2, \tag{8.58}$$

where π^{fb} is defined in (4.7) and such a solution exists in $(\hat{\pi},I)$ when $v'(\hat{\pi})>1+\tau>v'(I)$ as assumed in the text. In fact, we can without loss of generality choose

$$u_2^{fb}(\theta_1,\theta_2)=0,\quad \forall(\theta_1,\theta_2). \tag{8.59}$$

From (8.57), the firm's intertemporal participation constraint (4.5) is binding and thus

$$u_1^{fb}(\theta_1)=-\frac{\beta}{1-\beta}v(\pi^{fb}),\quad \forall\theta_1. \tag{8.60}$$

Because $\pi^{fb}>0$ and $v(0)=0$, we have

$$u_1^{fb}(\theta_1)<0,\quad \forall\theta_1. \tag{8.61}$$

‖

around $\hat{\pi}$, the firm is actually risk loving in that neighborhood as any profit level $\pi\in[0,\hat{\pi})$ can be transformed into a risky claim returning 0 with some probability ζ and $\hat{\pi}$ with complementary probability so that $\pi=(1-\zeta)\hat{\pi}$. Risk-neutral outsiders could sell this risky claim to the firm's owners at the sure price π. Buying such a claim would yield an expected payoff worth $(1-\zeta)v\left(\frac{\pi}{1-\zeta}\right)>v(\pi)=\pi$ to the firm. Generalizing this approach, we observe that, by buying risky claims, the firm can always concavify its payoff function so that its true utility function becomes the concave hull of v, namely $v^*=co(v)$. By doing so, the firm "becomes" risk neutral over a range of low profit levels. Firms with low levels of profit from the basic service then have a stochastic access to financial markets.

Proof. Proof of Lemma 5.1 and the general case in Section 7 The proof of Lemma 5.1 is a special case of the analysis in Section 7 and follows from the general proof when Δ is set equal to 0. We therefore only provide the proof in the general case where $\Delta \geq 0$.

The next lemma thus generalizes Lemma 5.1 to the case described in Section 7 where there is asymmetric information on the cost of the add-on. Of course, the second-period background risk induced by asymmetric information on the cost of the add-on also has an impact on first-period incentive compatibility. Our definition of the utility function w in the text in Section 7 indeed allows us to rewrite the requirement of incentive compatibility for a bidder with type θ_1 in terms of expected payoff as

$$\mathcal{U}(\theta_1) = \max_{\hat{\theta}_1 \in \Theta_1} (1-F(\hat{\theta}_1))^n \left((1-\beta)(b(\hat{\theta}_1)-\theta_1) + \beta w(b(\hat{\theta}_1)-\theta_1+y(\hat{\theta}_1),\Delta)\right). \tag{8.62}$$

The above maximum is achieved at θ_1, i.e.,

$$\mathcal{U}(\theta_1) = (1-F(\theta_1))^n ((1-\beta)u_1(\theta_1) + \beta w(\pi(\theta_1),\Delta)). \tag{8.63}$$

The second-period profit, $\pi(\theta_1) = u_1(\theta_1) + y(\theta_1)$ can now be expressed as

$$\pi(\theta_1) = \varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n} - (1-\beta)u_1(\theta_1)}{\beta}, \Delta\right) \tag{8.64}$$

where we define $\varphi(\zeta,\Delta)$ as the wealth level that guarantees ζ utils to the firm when the risk level is Δ, i.e., $\zeta = w(\varphi(\zeta,\Delta),\Delta)$.[31]

Lemma 8.3. Necessary condition. *Any incentive-compatible allocation $(\mathcal{U}(\theta_1), u_1(\theta_1))$ is such that $\mathcal{U}(\theta_1)$ is absolutely continuous in θ_1 (and thus almost everywhere differentiable) with at any point of differentiability:*

$$\dot{\mathcal{U}}(\theta_1) = -(1-F(\theta_1))^n \left(1-\beta+\beta w_\pi\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n} - (1-\beta)u_1(\theta_1)}{\beta}, \Delta\right), \Delta\right)\right). \tag{8.65}$$

Sufficient condition. *An allocation is incentive compatible if $\mathcal{U}(\theta_1)$ is absolutely continuous, satisfies (8.65) at any point of differentiability and is convex. A sufficient condition for this convexity is that $\pi(\theta_1)$ is non-decreasing.*

Integrating (8.65) and taking into account (5.12) yields the integral representation of the rent profile (7.31) (which simplifies to (5.13) when $\Delta = 0$).

Proof. Proof of Lemma 8.3 Necessity. From Theorem 2 and Corollary 1 in Milgrom and Segal (2002), it immediately follows that $\mathcal{U}(\theta_1)$ is absolutely continuous and thus almost everywhere differentiable with (8.65) holding at any point of differentiability.

31. The function φ is non-decreasing in both ζ and Δ. We have $\varphi_\zeta(\zeta,\Delta) = \frac{1}{w_\pi(\varphi(\zeta,\Delta),\Delta)} > 0$, and $\varphi_\Delta(\zeta,\Delta) = -\frac{w_\Delta(\varphi(\zeta,\Delta),\Delta)}{w_\pi(\varphi(\zeta,\Delta),\Delta)} > 0$.

SUFFICIENCY. For all $(\theta_1,\hat{\theta}_1)$, we rewrite (8.62) as

$$\begin{aligned}\mathcal{U}(\theta_1)\geq \mathcal{U}(\hat{\theta}_1)+(1-F(\hat{\theta}_1))^n\Big[&(1-\beta)(\hat{\theta}_1-\theta_1)\\ &+\beta\Big(w(\pi(\hat{\theta}_1)+(\hat{\theta}_1-\theta_1),\Delta)-w(\pi(\hat{\theta}_1),\Delta)\Big)\Big].\end{aligned}\tag{8.66}$$

Using (8.65) and absolute continuity, the rent profile $\mathcal{U}(\theta_1)$ satisfies the integral representation

$$\mathcal{U}(\theta_1)-\mathcal{U}(\hat{\theta}_1)=\int_{\theta_1}^{\hat{\theta}_1}(1-F(s))^n(1-\beta+\beta w_\pi(\pi(s),\Delta))ds,\quad \forall(\theta_1,\hat{\theta}_1)\in\Theta^2.$$

Condition (8.66) thus holds when

$$\int_{\theta_1}^{\hat{\theta}_1}(1-F(s))^n(1-\beta+\beta w_\pi(\pi(s),\Delta))ds\geq$$

$$(1-F(\hat{\theta}_1))^n\Big[(1-\beta)(\hat{\theta}_1-\theta_1)+\beta\Big(w(\pi(\hat{\theta}_1)+(\hat{\theta}_1-\theta_1),\Delta)-w(\pi(\hat{\theta}_1),\Delta)\Big)\Big].$$

Because w is concave in its first argument, we have $w(\pi(\hat{\theta}_1)+(\hat{\theta}_1-\theta_1),\Delta)-w(\pi(\hat{\theta}_1),\Delta)\leq (\hat{\theta}_1-\theta_1)w_\pi(\pi(\hat{\theta}_1),\Delta)$. A sufficient condition for (8.66) to hold is thus

$$\int_{\theta_1}^{\hat{\theta}_1}(1-F(s))^n(1-\beta+\beta w_\pi(\pi(s),\Delta))ds\geq(\hat{\theta}_1-\theta_1)(1-F(\hat{\theta}_1))^n\Big(1-\beta+\beta w_\pi(\pi(\hat{\theta}_1),\Delta)\Big).\tag{8.67}$$

Observe now that $(1-F(\theta_1))^n(1-\beta+\beta w_\pi(\pi(\theta_1),\Delta))$ non-increasing, i.e., $\mathcal{U}$ convex, implies (8.67) and thus (8.66). Finally, we compute

$$\ddot{\mathcal{U}}(\theta_1)=(1-F(\theta_1))^n\left(\frac{nf(\theta_1)}{1-F(\theta_1)}(1-\beta+\beta w_\pi(\pi(\theta_1),\Delta))-\beta w_{\pi\pi}(\pi(\theta_1),\Delta)\dot{\pi}(\theta_1)\right).$$

Because $w_{\pi\pi}\leq 0\leq w_\pi$, $\mathcal{U}$ is convex when $\dot{\pi}(\theta_1)\geq 0$. ‖

‖

Proof. Proof of Propositions 6.1 and 7.2 The proof of Proposition 6.1 is a special case of the proof of Proposition 7.2 where Δ is set to zero. We therefore only provide the general proof where $\Delta\geq 0$.

Due to the symmetry of bidders and the fact that the first-price auction always selects the firm with the lowest cost of the basic service, everything happens as if the principal was actually dealing with a single representative firm but this firm would have a cost for the basic service that would be drawn from the distribution of the minimum of $n+1$ independent variables drawn from the distribution F. The corresponding distribution function is thus $G(\theta_1)=1-(1-F(\theta_1))^{n+1}$ (with density

$g(\theta_1)=(n+1)f(\theta_1)(1-F(\theta_1))^n$). The principal's intertemporal payoff when dealing with the winning firm can thus be written as

$$\mathcal{W}(u_1(\theta_1),\mathcal{U}(\theta_1),\theta_1)=\mathcal{S}(\theta_1)-(1-\beta)u_1(\theta_1)-\beta(1+\tau)\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\right).$$

Given the type distribution G of the winning firm's cost as defined above, the problem with this representative firm can now be written as follows:

$$(\mathcal{P}^{as}):\qquad \max_{(u_1(\theta_1),\mathcal{U}(\theta_1),\tilde{\theta}_1)}\int_{\underline{\theta}_1}^{\tilde{\theta}_1}\mathcal{W}(u_1(\theta_1),\mathcal{U}(\theta_1),\theta_1)g(\theta_1)d\theta_1 \text{ subject to (8.65) and (5.12).}$$

This is a relaxed optimization problem since the requirement for incentive compatibility has been reduced to the necessary condition (8.65) in Lemma 8.3. The solution $\pi^{sb}(\theta_1,\Delta)$ to this relaxed problem is actually the solution to the original problem when $\pi^{sb}(\theta_1,\Delta)$ is non-decreasing as proved in Lemma 8.3. We will check this monotonicity condition towards the end of this proof.

Equipped with the above expression of the principal's problem ($\mathcal{P}^{as}$), and denoting by λ the co-state variable for (8.65), we can write the Hamiltonian for this optimization problem as

$$\mathcal{H}(u_1,\mathcal{U},\lambda,\theta_1)=(n+1)f(\theta_1)(1-F(\theta_1))^n\mathcal{W}(u_1,\mathcal{U},\theta_1) \tag{8.68}$$
$$-\lambda(1-F(\theta_1))^n\left(1-\beta+\beta w_\pi\left(\varphi\left(\frac{\frac{\mathcal{U}}{(1-F(\theta_1))^n}-(1-\beta)u_1}{\beta},\Delta\right),\Delta\right)\right).$$

We can use the Pontryagin Principle to get necessary conditions for optimality. (See Ch. 2, Th. 2 in Seierstad and Sydsæter (1987).) Provided that asymmetric information on θ_1 is not too large (as assumed in the text), the solution $\pi^{sb}(\theta_1,\Delta)$ remains close to π^{fb} and this optimum is obtained on the range where w and w_π are both differentiable in π. The corresponding necessary conditions are listed below.

• *Costate variable.* There exists λ, continuous and piecewise differentiable, such that

$$\dot{\lambda}(\theta_1)=\left((1+\tau)(n+1)f(\theta_1)+\lambda(\theta_1)w_{\pi\pi}\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\right),\Delta\right)\right)\times \tag{8.69}$$
$$\varphi_\zeta\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\right).$$

• *Transversality condition.* Because there is no boundary condition on $\mathcal{U}$ at $\underline{\theta}$, the transversality condition is given by

$$\lambda(\underline{\theta})=0. \tag{8.70}$$

•*Optimality condition with respect to u_1.* Using a first-order condition with respect to u_1, we find

$$\frac{(n+1)f(\theta_1)}{\varphi_\zeta\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\right)}=(1+\tau)(n+1)f(\theta_1)+\lambda(\theta_1)w_{\pi\pi}\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\right),\Delta\right). \tag{8.71}$$

We now use those optimality conditions to derive more specific results.

• *Propositions 6.1 and 7.2.* Inserting (8.71) into (8.69) yields

$$\dot{\lambda}(\theta_1)=(n+1)f(\theta_1). \tag{8.72}$$

Taking into account (8.70) now gives

$$\lambda(\theta_1)=(n+1)F(\theta_1). \tag{8.73}$$

Inserting this expression into (8.71), and simplifying yields (7.33). The condition (6.16) is obtained when $\Delta=0$ which is akin to the case where θ_2 is verifiable.

• *Monotonicity condition.* Straightforward differentiation of (7.33) yields

$$\dot{\pi}^{sb}(\theta_1)=\frac{\frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right)w_{\pi\pi}(\pi^{sb}(\theta_1,\Delta),\Delta)}{w_{\pi\pi}(\pi^{sb}(\theta_1,\Delta),\Delta)-\frac{F(\theta_1)}{f(\theta_1)}w_{\pi\pi\pi}(\pi^{sb}(\theta_1,\Delta),\Delta)}. \tag{8.74}$$

That v is concave implies that $w_{\pi\pi}\leq 0$, while Assumption (2.1) means that the derivative of the hazard rate is non-negative which together with Assumption (2.1) implies that the numerator on the right-hand side of (8.74) is non-positive. Finally, $v'''\geq 0$ implies $w_{\pi\pi\pi}\geq 0$ and yields the required monotonicity result that is used both in Proposition 6.1 and in Proposition 7.2. Hence, $\mathcal{U}^{sb}(\theta_1)$ is convex and the sufficiency condition for Lemma 5.1 holds when Assumption (2.1) holds and $w_{\pi\pi\pi}\geq 0$.

• *Upper bound on $\pi^{sb}(\theta_1)$.* We just saw that v concave implies $w_{\pi\pi}\leq 0$, while $v'''\geq 0$ implies $w_{\pi\pi\pi}\geq 0$. Hence $w_{\pi\pi}-\frac{F(\theta_1)}{f(\theta_1)}w_{\pi\pi\pi}\leq 0$. Therefore, $\pi^{sb}(\theta_1,\Delta)$ as defined from the first-order condition (7.33) is less than I when $w_\pi(I,\Delta)<1+\tau+\frac{F(\theta_1)}{f(\theta_1)}w_{\pi\pi}(I,\Delta)$, which, thanks to Assumption (2.1), is implied by (7.32). Condition (6.15) plays the same role when θ_2 is verifiable (i.e., $\Delta=0$).

•*Comparison between the case θ_2 verifiable and the case θ_2 private information.* Observe that $v''''\leq 0$ implies $w_{\pi\pi\Delta}\leq 0$. From (7.33), we thus deduce

$$w_\pi(\pi^{sb}(\theta_1,\Delta),0)-\frac{F(\theta_1)}{f(\theta_1)}w_{\pi\pi}(\pi^{sb}(\theta_1,\Delta),0)=v'(\pi^{sb}(\theta_1,\Delta))-\frac{F(\theta_1)}{f(\theta_1)}v''(\pi^{sb}(\theta_1,\Delta))\leq 1+\tau,\quad \forall\theta_1\leq\tilde{\theta}_1. \tag{8.75}$$

Comparing with (6.16) and using that $v'(\pi)-\frac{F(\theta_1)}{f(\theta_1)}v''(\pi)$ is non-increasing in π when $v'''\geq 0$ yields (7.34).

• *Comparison with the case of full information.* Consider first the case where θ_2 is verifiable. It immediately follows from (6.16) that $1+\tau\geq v'(\pi^{ie}(\theta_1)),\forall\theta_1\leq\tilde{\theta}_1$, which amounts to the left-hand side inequality in (6.17). ‖

Proof. Proof of Propositions 6.2 and 7.3 We start with the general case $\Delta \geq 0$ (relevant for Proposition 7.3) and specialize our analysis later to the case $\Delta = 0$ (Proposition 6.2). We first come back to the expression of the Hamiltonian $\mathcal{H}$ as defined in (8.68). The necessary condition for optimality with respect to the free-end point $\tilde{\theta}_1$ (Theorem 11, p. 145, Seierstad and Sydsæter (1987)) when at an interior point writes as $\mathcal{H}(u_1^{sb}(\tilde{\theta}_1^{sb}), \mathcal{U}^{sb}(\tilde{\theta}_1^{sb}), \lambda(\tilde{\theta}_1^{sb}), \tilde{\theta}_1^{sb}) = 0$. This necessary condition can be rewritten as

$$\mathcal{W}(u_1^{sb}(\tilde{\theta}_1^{sb}), \mathcal{U}^{sb}(\tilde{\theta}_1^{sb}), \tilde{\theta}_1^{sb}) = \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(1-\beta+\beta w_\pi(\pi^{sb}(\tilde{\theta}_1^{sb}), \Delta)).$$

Replacing $\mathcal{W}$ by its full expression and rearranging terms yields

$$\mathcal{S}(\tilde{\theta}_1^{sb}) = (1-\beta)u_1^{sb}(\tilde{\theta}_1^{sb}) + \beta(1+\tau)\varphi\left(-\frac{1-\beta}{\beta}u_1^{sb}(\tilde{\theta}_1^{sb}), \Delta\right) + \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(1-\beta+\beta w_\pi(\pi^{sb}(\tilde{\theta}_1^{sb}), \Delta)). \tag{8.76}$$

Using the definition of φ and (5.12), we have

$$\beta w\left(\varphi\left(-\frac{1-\beta}{\beta}u_1^{sb}(\tilde{\theta}_1^{sb}), \Delta\right), \Delta\right) = -(1-\beta)u_1^{sb}(\tilde{\theta}_1^{sb}).$$

Inserting into (8.76) and using the definition of $\pi(\tilde{\theta}_1^{sb})$ from (8.64) yield (6.25) (and (6.20) in the case $\Delta = 0$).

Conditions for an interior solution. Taking into account condition (6.25), an interior solution $\tilde{\theta}_1^{sb}$ exists when, first

$$\mathcal{S}(\underline{\theta}) + \beta\left(w(\pi^{fb}(\Delta), \Delta) - (1+\tau)\pi^{fb}(\Delta)\right) > 0,$$

where $\pi^{fb}(\Delta)$ solves $w_\pi(\pi^{fb}(\Delta), \Delta) = 1+\tau$ and, second

$$\mathcal{S}(\overline{\theta}) + \beta\left(w(\pi^{sb}(\overline{\theta}, \Delta), \Delta) - (1+\tau)\pi^{sb}(\overline{\theta}, \Delta)\right) < \frac{1}{f(\overline{\theta})}(1-\beta+\beta w_\pi(\pi^{sb}(\overline{\theta}, \Delta), \Delta))$$

where $\pi^{sb}(\overline{\theta}, \Delta)$ follows from (7.33).

The case where θ_2 is verifiable. Replacing Δ by 0 into (8.76) yields the expression for an interior solution $\tilde{\theta}^{ie}$ given in (6.20).

Comparison of the participation thresholds. Consider the function

$$\Psi(\theta_1, \Delta) = w(\pi^{sb}(\theta_1, \Delta), \Delta) - (1+\tau)\pi^{sb}(\theta_1, \Delta) - \frac{F(\theta_1)}{f(\theta_1)}w_\pi(\pi^{sb}(\theta_1, \Delta), \Delta).$$

Using that $\pi^{sb}(\theta_1, \Delta)$ satisfies (7.33), we compute

$$\Psi_\Delta(\theta_1, \Delta) = w_\Delta(\pi^{sb}(\theta_1, \Delta), \Delta) - \frac{F(\theta_1)}{f(\theta_1)}w_{\pi\Delta}(\pi^{sb}(\theta_1, \Delta)), \Delta) < 0 \tag{8.77}$$

where the right-hand side inequality follows from $w_\Delta < 0$ (which follows from $v'' \leq 0$) and $w_{\pi\Delta} \geq 0$ (which follows from Corollary 7.1). Hence, $\Psi(\theta_1, \Delta) < \Psi(\theta_1, 0) = v(\pi^{ie}(\theta_1)) - (1+$

$\tau)\pi^{ie}(\theta_1) - \frac{F(\theta_1)}{f(\theta_1)} v'(\pi^{ie}(\theta_1))$. From this it follows that

$$\mathcal{S}(\tilde{\theta}_1^{sb}) + \beta\left(v(\pi^{ie}(\tilde{\theta}_1^{sb})) - (1+\tau)\pi^{ie}(\tilde{\theta}_1^{sb})\right) > \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(1-\beta+\beta v'(\pi^{ie}(\tilde{\theta}_1^{sb}))).$$

Provided that the necessary condition for optimality is also sufficient, this latter inequality implies (7.38). ‖

Proof. Proof of Corollary 6.3 With frictionless finance, the expected efficiency gains from running both the basic service over two periods and an innovative add-on overall amounts to

$$\int_{\underline{\theta}_1}^{\tilde{\theta}_1} g(\theta_1)(\mathcal{S}(\theta_1) + \beta(1+\tau)(R(e^{fb}) - I))d\theta_1.$$

Given the expression of the firm's information rent $\mathcal{U}(\theta_1)$ given in (5.14), the optimal reserve price $\tilde{\theta}_1^{rn}$ maximizes

$$\int_{\underline{\theta}_1}^{\tilde{\theta}_1} g(\theta_1)\left(\mathcal{S}(\theta_1) + \beta(1+\tau)(R(e^{fb}) - I) - \frac{1}{(1-F(\theta_1))^n}\int_{\theta_1}^{\tilde{\theta}_1}(1-F(s))^n ds\right)d\theta_1. \quad (8.78)$$

The first-order necessary condition for optimality immediately gives us the expression for an interior cutoff $\tilde{\theta}_1^{rn}$ as in (8).

Because $v' \geq 1$ over the range $[\hat{\pi}, I)$, we can provide a lower bound on the right-hand side of (6.20) as

$$\mathcal{S}(\tilde{\theta}_1^{ie}) + \beta\left(v(\pi(\tilde{\theta}_1^{ie})) - (1+\tau)\pi(\tilde{\theta}_1^{ie})\right) \geq \frac{F(\tilde{\theta}_1^{ie})}{f(\tilde{\theta}_1^{ie})}. \quad (8.79)$$

Now, observe that v concave implies

$$v(\pi(\tilde{\theta}_1^{ie})) \leq v(\pi^{fb}) + v'(\pi^{fb})(\pi(\tilde{\theta}_1^{ie}) - \pi^{fb}).$$

Using (4.7), yields

$$v(\pi(\tilde{\theta}_1^{ie})) - (1+\tau)\pi(\tilde{\theta}_1^{ie}) \leq v(\pi^{fb}) - (1+\tau)\pi^{fb}.$$

Because $v(\pi^{fb}) \leq R(e^{fb}) - I + \pi^{fb}$, we thus have

$$v(\pi(\tilde{\theta}_1^{ie})) - (1+\tau)\pi(\tilde{\theta}_1^{ie}) \leq R(e^{fb}) - I - \tau\pi^{fb} \leq (1+\tau)(R(e^{fb}) - I)$$

where the last inequality follows from $R(e^{fb}) - I + \pi^{fb} \geq R(e^{fb}) - I \geq 0$. Hence, the social benefit on the left-hand side of (6.20) is bounded above by

$$\mathcal{S}(\tilde{\theta}_1^{ie}) + \beta(1+\tau)(R(e^{fb}) - I). \quad (8.80)$$

Combining (8.79) and (8.80) yields

$$\mathcal{S}(\tilde{\theta}_1^{ie}) + \beta(1+\tau)(R(e^{fb}) - I) \geq \frac{F(\tilde{\theta}_1^{ie})}{f(\tilde{\theta}_1^{ie})}. \quad (8.81)$$

Provided that the necessary condition for optimality in (8.78) is also sufficient, (8) and (8.81) together imply (6.22). ‖

Proof. Proof of Proposition 6.4 Using the expressions in (5.9) and (5.13) yields (6.23).

To provide conditions such that $b^{ie}(\theta_1)$ is indeed increasing, we differentiate (6.23) and obtain

$$\dot{b}^{ie}(\theta_1)=\frac{-\beta}{1-\beta}v'(\pi^{ie}(\theta_1))\left(\dot{\pi}^{ie}(\theta_1)+1\right)+\frac{nf(\theta_1)}{1-F(\theta_1)}\frac{1}{(1-F(\theta_1))^n}\int_{\theta_1}^{\tilde{\theta}_1}(1-F(s))^n\left(1+\frac{\beta}{1-\beta}v'(\pi(s))\right)ds.$$

The first term is negative since from Proposition 6.1, $\pi^{ie}(\theta_1)$ is non-decreasing. A sufficient condition for an increasing equilibrium bidding strategy to exist is thus β sufficiently small. In the example of costly finance a sufficient condition for an increasing bidding strategy to exist can be deduced from (6.25) and requires that $\beta \leq \frac{n}{2(n+1)}$. ‖

Proof. Proof of Corollary 7.1 We have $w_{\pi\Delta}(\pi,\Delta)=\nu(1-\nu)(v''(\pi+(1-\nu)\Delta)-v''(\pi-\nu\Delta))\geq 0$ when both $\pi+(1-\nu)\Delta\in[\hat{\pi},I)$ and $\pi-\nu\Delta\in[\hat{\pi},I)$ since $v'''\geq 0$ on this range. ‖

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10. DATA AVAILABILITY STATEMENT

No new data were generated or analysed in support of this research.

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