Bargaining as a Struggle Between Competing Attempts at Commitment *

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Abstract

The strategic importance of commitment in bargaining is widely acknowledged. Yet disentangling its role from key features of canonical models, such as proposal power and reputational concerns, is difficult. This paper introduces a model of bargaining with strategic commitment at its core. Following Schelling (1956), commitment ability stems from the costly nature of concession and is endogenously determined by players' demands. Agreement is immediate for familiar bargainers, modelled via renegotiation-proofness. The unique prediction at the high concession cost limit provides a strategic foundation for the Kalai bargaining solution. Equilibria with delay feature a form of gradualism in demands.
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1 Introduction

If two agents seek to divide some surplus, what division will they agree on and when and how? This set of questions, that I collectively label the bargaining problem, is key to a vast range of economic interactions. Economic models rely on the strategic theory of bargaining to resolve it, either directly or indirectly by informing the appropriate choice of a bargaining solution.

Strategic models of bargaining that allow negotiations to unfold over time typically have at their core either the alternating-offers model of Rubinstein (1982) or the reputation model of Abreu and Gul (2000). Schelling (1956, 1960) proposes a third approach. As summarized in Crawford (1982), Schelling views the bargaining process as a struggle between players to commit themselves to —that is, to convince their opponent of their inability to retreat from —advantageous bargaining positions. Schelling's own treatment of his approach was impressionistic and by way of examples. Subsequent work has either developed the theory in static environments or focused on evaluating the role of commitment while relying on one of the two canonical models mentioned above to resolve the underlying bargaining problem.[1]

This paper presents a formalization of Schelling's theory with an infinite-horizon model of bargaining with complete information. The objective is to characterize the extent to which this theory, built on the use of strategic commitments alone, resolves the bargaining problem and how, and furthermore establish conditions under which the model's predictions are adequately summarized by some bargaining solution.

The model builds on two key elements of Schelling's theory. First, a bargainer may find it costly to back down from a stated demand and this is the source of her commitment ability. Second, the commitment ability is nevertheless endogenous, in that it depends on the demands. A less aggressive demand weakens the opponent's commitment ability by allowing more room for her to back down. By contrast, a demand that leaves an opponent's back against the wall only

[1] See for example, Crawford (1982), Muthoo (1996), Ellingsen and Miettinen (2008) and Dutta (2012) for the first and Fershtman and Seidmann (1993), Compte and Jehiel (2004), Wolitzky (2012) and Basak and Deb (2020) for the second. Ellingsen and Miettinen (2014) consider a dynamic model of a hybrid nature that I discuss in detail in section 5.4.

ensures the latter's commitment.

In the model, the bargainers simultaneously announce demands. If the demands are compatible, bargaining ends on those terms. If incompatible, the players decide whether to stick to their demand or concede to the opponent's offer. Concession incurs an additional cost which is increasing in the conceded amount. If neither player concedes, then the current period of bargaining ends and the next period begins with a fresh round of demands. The game proceeds in this manner until either compatible demands or a concession following incompatible demands. The bargainers are impatient, as captured by constant discount factors. I focus on subgame perfect equilibria with pure strategies in the demand stage (henceforth SPE).

The model can be seen as a variant of the infinite horizon version of the Nash Demand Game (henceforth IH-NDG). While in the latter, incompatible demands end the current round of bargaining, in the present model bargainers get a chance to concede. Indeed, if the concession costs are made arbitrarily high, then concession is effectively ruled out and the IH-NDG obtains at the limit.

The model predictions depend on two sets of parameters, namely the discount factors and concession cost functions. In any SPE outcome, the bargainers eventually agree upon an efficient division of the surplus, following some delay, if any. In contrast to common dynamic bargaining models, the range of efficient divisions of the surplus that can arise in equilibrium is linked to the maximum delay the equilibrium accommodates following any history. Delay, while permitted under SPE, has an upper bound.

Renegotiation-proof SPE, used to model familiar bargainers, feature no delay and an exact characterization obtains for the corresponding set of surplus divisions. This leads to a key finding of the paper. As the marginal concession costs are made arbitrarily high, the set of renegotiation-proof SPE outcomes converges to selecting a unique efficient outcome in the limiting IH-NDG. This outcome is identical to that of the Kalai bargaining solution (see Kalai (1977)) with its proportion determined by the discount factors and a limit ratio of the concession cost functions. Therefore, not only does the formalization of Schelling's theory fully resolve the bargaining problem, it also provides a strategic foundation for the Kalai bargaining solution. Furthermore the parameters of the non-cooperative model select the appropriate bargaining solution from the family of solutions characterized in Kalai (1977).

Markov perfect equilibria (which may violate renegotiation-proofness) can exhibit delay. In a natural way, such equilibria with delay yield a form of gradualism, the feature in which bargainers start with extreme demands that soften over time. Finally (and surprisingly), the set of stationary Markov perfect equilibrium *outcomes* coincides with the set of renegotiation-proof SPE outcomes, despite the latter allowing arbitrarily history dependent strategies.

As Binmore, Osborne and Rubinstein (1992) states, *The ultimate aim of what is now called the "Nash program" (see Nash 1953) is to classify the various institutional frameworks within which negotiation takes place and to provide a suitable "bargaining solution" for each class.* This paper contributes to this literature by making a case for the Kalai bargaining solution in environments in which commitment ability due to concession costs is salient.[2] Binmore, Rubinstein and Wolinsky (1986) establish a robust connection between the alternating-offers model and the Nash bargaining solution. Studies on commitment that rely on the alternating-offers model, such as Muthoo (1996), find similar support for the (asymmetric) Nash bargaining solution. Relying on the struggle to commit itself to resolve the bargaining problem, as the current paper shows, leads instead to the Kalai bargaining solution. This is an important distinction. The appropriate choice of a bargaining solution is not merely a game-theoretic curiosity. Aruoba, Rocheteau and Waller (2007), for instance, show that the choice of bargaining solution matters both qualitatively and quantitatively for questions of first-order importance in monetary economics.

To the best of my knowledge, Dutta (2012) and Hu and Rocheteau (2020) are the only other papers that provide strategic bargaining foundations for the Kalai bargaining solution. Hu and Rocheteau (2020) rely on the alternating-offers model. They show that if the surplus is divided into N parts and in each of N rounds players engage in Rubinstein bargaining over one of these parts,

[2] Examples of such environments are described in section 5.1.

then the outcome corresponds to the Kalai bargaining solution as N tends to infinity. While theoretically insightful, the procedure with large N is difficult to descriptively align with typical bargaining narratives.

Dutta (2012) is the static (one-period) version of the current model and captures a qualitatively similar role for the concession costs, in that higher costs benefit the bargainer. It shares the unrealistic feature of the Nash demand game in ruling out future negotiations following a single round of disagreement, and as a result has no role for discount factors.

Given the limit uniqueness result of the static model in Dutta (2012), it is natural to expect (with some work) a similar result in the dynamic model under stationary strategies. A novel and surprising finding in the current paper is that the Kalai solution arises as the unique limit even under the assumption of renegotiation-proofness, which allows for arbitrarily history dependent strategies.[3] There is no reason to expect renegotiation-proof outcomes to coincide with stationary ones in dynamic bargaining games. Indeed, as discussed in 3.2, the acute multiplicity (of surplus division outcomes) in the multilateral version of the Rubinstein bargaining game persists unabated under the assumption of renegotiation-poofness, while stationarity delivers a unique result. The finding that the two sets of outcomes coincide in the current model is the result of the specific structure of its SPE, discussed in section 2.1.

The rest of the paper is as follows. In section 2, I introduce the general model and show how all SPE have a simple structure. In section 3, I focus on a linear specification. In this setting, characterization results under subgame perfection, renegotiation-proofness and Markov perfection are obtained in subsections 3.1, 3.2 and 3.3, respectively. In section 4, I return to the general model, characterize SPE outcomes categorized by maximum permissible delay, including the set of renegotiation-proof SPE outcomes and establish the link with the Kalai solution. A discussion of the intuition follows. In section 5, I discuss some key features of the model and other related literature. Proofs of all results are collected in the appendix, unless stated in the main text.

[3] The intuition behind this is discussed in section 4.2.

2 The Model

Two players, 1 and 2, play an infinite horizon game to split a pie of size 1. In period $t \in \mathbb{N} \equiv \{1, 2, 3, \ldots\}$, if the bargaining problem is still unresolved, each player $i \in \{1, 2\}$ announces a demand $z_i \in [0, 1]$. The announcements are simultaneous. For a given demand profile $z = (z_1, z_2)$, let $d(z) = z_1 + z_2 - 1$. If the demands are compatible ($d(z) \leq 0$) then the game ends with both players receiving their own demands. The resulting payoff profile is $(u_1(z_1), u_2(z_2))$, where u_i is the payoff function for player i.

Following incompatible demands ($d(z) > 0$), the bargainers enter a concession stage. Here the players simultaneously decide whether to stick to their demands or back down and accept the other's offer. Backing down comes at a cost which is a function of the conceded amount, the difference between the initial demand and the accepted amount, $z_i - (1 - z_{-i}) = d(z)$, and is captured by the *concession cost function* c_i. If both players stick to their demand then the bargaining problem remains unresolved and moves to the next period. This concession stage game is represented in the table below.

Table 1: Concession Stage following Incompatible Demand Profile z

	Accept (A)	*Stick (S)*
A	$u_1(1-z_2) - c_1(d(z)), u_2(1-z_1) - c_2(d(z))$	$u_1(1-z_2) - c_1(d(z)), u_2(z_2)$
S	$u_1(z_1), u_2(1-z_1) - c_2(d(z))$	$u_1(0), u_2(0)$

As long as some player chooses A the game ends this period with the associated payoffs in the table, otherwise it moves to period $t + 1$.[4] The following assumptions hold throughout the paper.

Assumption 1 *For $i \in \{1, 2\}$, $u_i : [0, 1] \to \mathbb{R}_+$ is a strictly increasing, concave and continuously differentiable function with $u_i(0) = 0$.*

Assumption 2 *For $i \in \{1, 2\}$, $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing, unbounded above and continuously differentiable function with $c_i(0) = 0$.*

[4]Following AA in the concession stage, $d(z)$ is left on the table. Alternative specifications of this outcome that split $d(z)$ between the bargainers in some way leave all results unchanged.

A history of play that leads to the beginning of period $t+1$ with $t \in \mathbb{N}$, denoted as h^t, is a sequence of t incompatible demand profiles with (S, S) in the corresponding concession stages, $(z^1, SS, z^2, SS, \ldots, z^t, SS)$. Let H^t be the set of all such t-period histories, with the null history $H^0 = \{h^0\}$ and $H = \cup_{t=0}^{\infty} H^t$. A history of play that leads to the concession stage in period t, denoted as $h^{t'}$, is an element of H^{t-1} followed by an incompatible demand profile z^t. Let $H^{t'}$ be the set of all such t-period histories and $H' = \cup_{t=1}^{\infty} H^{t'}$. A pure strategy for player i is a function $\sigma_i : H \cup H' \rightarrow [0,1] \cup \{A, S\}$ such that $\sigma_i(h) \in [0,1]$ for $h \in H$ and $\sigma_i(h) \in \{A, S\}$ for $h \in H'$. The subgame following history $h \in H \cup H'$ is labeled $g(h)$.

Given a history $h^t \in H$, a strategy profile $\sigma = (\sigma_1, \sigma_2)$ determines the period $n > t$ when bargaining ends in the subgame $g(h^t)$, with payoffs in that period of $y = (y_1, y_2)$, where $y = (0,0)$ if $n = \infty$.[5] Call $(y, n-t)$ the outcome of the game $g(h^t)$ under σ. A strategy profile σ with outcome $(y, n-t)$ in the subgame $g(h^t)$ yields the discounted payoff of $\delta_i^{n-t-1} y_i$ to player i at the beginning of the subgame, where $\delta_i \in (0,1)$ is player i's discount factor.

2.1 Preliminaries

To analyze its content, I focus on *pure strategy subgame perfect equilibria* of the model. Subsequently, for expositional ease, I refer to these simply as subgame perfect equilibria or SPE. Infinite horizon games with simultaneous moves typically feature a vast multiplicity of SPE with a sense of *anything goes*. The current model features multiplicity too. Nevertheless, the following straightforward yet useful lemma shows that all such equilibria have a simple structure. Exactly compatible demands imply $d(z) = 0$.

Lemma 1 *A subgame perfect equilibrium at any period must feature either*
(a) exactly compatible demands, or
(b) incompatible demands followed by both players choosing Stick*.*

Proof. Consider a period in which incompatible demands (z) are followed by some action profile other than (S, S) in the concession stage. Then, as the

[5] Notice that y_i, which denotes i's payoff in the period when bargaining ends, is distinct from the payoff function u_i. If i receives z_i without making a concession then $y_i = u_i(z_i)$. With a concession, $y_i = u_i(z_i) - c_i(d(z))$.

payoff matrix in table 1 shows, there must be some player i who receives a payoff strictly less than $u_i(1 - z_{-i})$ and is strictly better off by deviating to the compatible demand $1 - z_{-i}$ instead of the original z_i.

Next, given a period with compatible demands that add up to less than 1, the player with the lower demand, say i, is strictly better off demanding $1 - z_{-i}$ instead. ■

In other words, any SPE involves some rounds of delay, if any, via incompatible demands, followed by an agreement on an efficient division of the surplus. Therefore the two components of any SPE outcome (y, n) correspond to an (eventual) compatible profile z where $y = u(z)$ and delay $n - 1 \geq 0$.

Subgame Perfect Equilibrium with Maximum Delay m

Dynamic bargaining games with multiple SPE typically have the following feature.[6] The range of efficient SPE outcomes constitutes the first-order multiplicity. These rely on history-dependent strategies but do not require strategy profiles involving delay. This first-order multiplicity is used, through appropriate history-dependent strategies, to generate varying lengths of delay, the second-order multiplicity. In the current model, delay is on a more equal footing with the set of efficient SPE outcomes. Limiting the length of delay permissible in an SPE limits the range of efficient outcomes that can arise in equilibrium. The following classification of SPEs permits partial characterization results that demonstrate this feature.

Definition 1 *An SPE σ is called an SPE with maximum delay m if for any subgame $g(h^t)$, $h^t \in H$, it generates an outcome $(y, n - t)$ where $n - t - 1 \leq m$.*

The classification is particularly useful because in the absence of an exact characterization of SPE outcomes, it permits a simple yet sharper partial characterization of equilibrium outcomes compared to SPE alone. Further, it is a consistent behavioural restriction, since it imposes a bound on delay both on and off the equilibrium path. Indeed, it is a natural generalization of the commonly studied no-delay equilibrium, which requires SPE to feature no delay following

[6] See, for instance, Sutton (1986), Avery and Zemsky (1994) and Merlo and Wilson (1995).

any history.[7]

The results that follow rely on the stationary structure of the model. To this end, for any $h \in H$, let $O^m(h)$ denote the set of outcomes of SPE with maximum delay m in the subgame $g(h)$. By Lemma 1 any SPE outcome (y, n) corresponds to an (eventual) compatible profile z where $y = u(z)$ and delay $n - 1 \geq 0$. Now define

$$B^m \equiv \left\{ z | (u(z), t) \in O^m(h^0) \right\}$$

to be the set of all efficient surplus divisions (exactly compatible profiles) that can arise as the outcome of some SPE with maximum delay m in the bargaining game. Due to the stationary structure of the game and definition 1, it follows that $B^m = \{z | (u(z), t) \in O^m(h)\}$, for all $h \in H$.

3 The Linear Model

In this section, I analyze the following specification of the bargaining model.

$$\forall i \in \{1, 2\}, \qquad u_i(z_i) = z_i \qquad \textit{and} \qquad c_i(d(z)) = k_i d(z) \textit{ for some } k_i > 0.$$

This linear specification retains the strategic tradeoffs of the general model while allowing closed-form characterizations of equilibrium outcomes.

3.1 Subgame Perfection

The reason why subgame perfection rules some compatible demands out and others in, lies in the concession stage behaviour permitted in equilibrium. Consider period 1 of the game, assuming that any impasse leads to a continuation (present-discounted) payoff profile, w. Then, the augmented concession game following an incompatible profile z is described in table 2.

Table 2: Augmented Concession Game following Incompatible Profile z

	A	S
A	$1 - z_2 - k_1(z_1 + z_2 - 1), 1 - z_1 - k_2(z_1 + z_2 - 1)$	$1 - z_2 - k_1(z_1 + z_2 - 1), z_2$
S	$z_1, 1 - z_1 - k_2(z_1 + z_2 - 1)$	w_1, w_2

Each incompatible profile z leads to one of four distinct equilibrium scenarios

[7] See Ray and Vohra (2015), Collard-Wexler, Gowrisankaran and Lee (2019) and Brugemann, Gautier and Menzio (2019).

in the augmented concession game. Three of these involve a unique dominance solvable outcome (i) (S,A), (ii) (S,S) and (iii) (A,S). In the fourth, both (S,A) and (A,S) are Nash equilibria. These regions are depicted in figure 1.

Figure 1 depicts both the space of demands and present-discounted payoffs of the two players. Exactly compatible demand profiles lie on AB. The rest of ABC contains all feasible incompatible demand profiles. Due to discounting, all present-discounted payoffs lie on or in the triangle AOB.

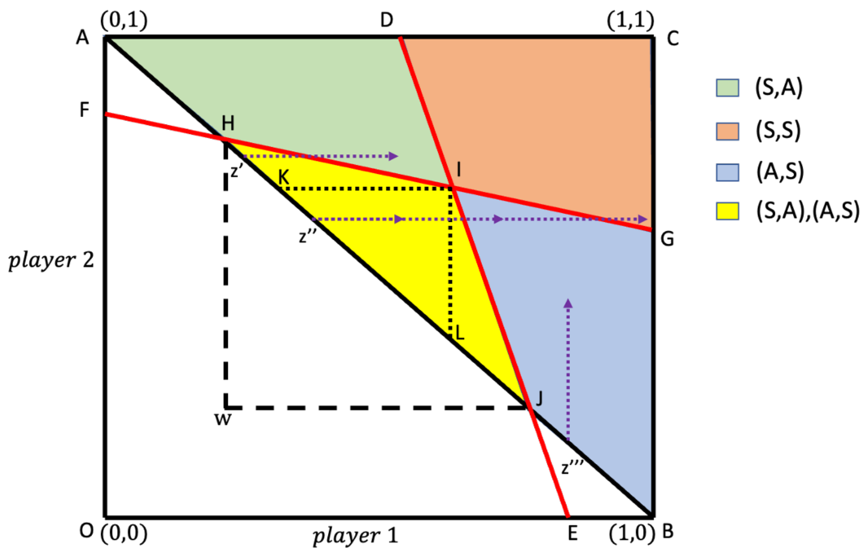


Figure 1

The coloured regions are pinned down by the two red lines DE and FG, which map the equations $1 - z_1 - k_2(z_1 + z_2 - 1) = w_2$ and $1 - z_2 - k_1(z_1 + z_2 - 1) = w_1$, respectively, labelled *indifference lines*. Following an incompatible profile on DE, in the concession game, player 2 is indifferent between the outcomes (S, A) and (S, S).[8] So conditional on player 1's choice of S, player 2 would stick to her demand following incompatible demand profiles "above" DE and back down following those "below" DE. FG is the analogous line for player 1.

Consequently, for instance, following an incompatible demand profile in the green region, and with a continuation payoff profile w, the concession stage

[8]In the concession stage, either player always strictly prefers S over A conditional on the other player choosing A.

features a unique equilibrium (dominance solvable) outcome; player 1 sticks to her demand while player 2 backs down, (S, A).

Next notice that compatible demand profiles from the interval AK cannot arise in period 1 in equilibrium, since player 1 can profitably deviate into the green region, as depicted for the profile z' (by the arrow). Compatible profiles in LB are similarly ruled out, since player 2 can now profitably deviate to the blue region, as depicted for profile z'''. By contrast, the multiplicity of equilibria in the yellow region can be used to rule out such deviations for profiles in KL.[9]

Consider a strategy profile in which any deviation from a compatible demand profile *into* the yellow region, is followed by the Nash equilibrium in the (augmented) concession stage wherein the deviator chooses A while the other player chooses S. With such a strategy profile, no player has an incentive to deviate from a compatible demand in KL. Take z'' in the figure, for instance. The arrows show some deviation options for player 1, none of which are profitable. Deviating to the yellow region requires player 1 to then back down. The same holds for deviation to the blue region, where (A, S) is the dominance solvable outcome. Deviating to the orange region leads to the dominance solvable outcome of (S, S) with a payoff of w_1. Likewise, player 2 can only deviate to the yellow and green regions, neither of which is profitable. All compatible profiles in KL can similarly arise in equilibrium, given the continuation profile w.

Whenever Player 2(1) can deviate to the blue (green) region from some compatible profile, she strictly prefers to do so. However, the blue (green) region is a function of the continuation payoff, and may vary with the demand profile in an SPE. But if player 2(1) has a deviation from compatible profile z that lies in a blue (green) region for *every possible equilibrium continuation payoff*, then z cannot arise in equilibrium.[10] The following lemma states this restriction on equilibrium compatible demand profiles for SPE with maximum delay n^*. The proof in the appendix establishes the argument above formally.

[9] The compatible profiles at K and L are the ones in which players 2 and 1, respectively, receive their share from the (incompatible) profile at I.

[10] In other words, equilibrium compatible demands must be such that neither bargainer can raise her own demand and extract a concession from her opponent. The latter requires the (unique) dominance solvable outcome of the concession game to have the deviator stick to her demand while her opponent concedes, irrespective of the equilibrium continuation play.

Lemma 2 *Suppose σ is a pure strategy profile with $\sigma(h^{t-1}) = z$ and $\sum_{i=1}^{2} z_i = 1$ for some $h^{t-1} \in H$. If for some $i \in \{1,2\}$, there exists $z_{-i} < \hat{z}_{-i} \leq 1$ such that*

$$1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^{n} \tilde{z}_{-i} \tag{1}$$

and

$$1 - \hat{z}_{-i} - k_i(z_i + \hat{z}_{-i} - 1) > \delta_i^n \tilde{z}_i \tag{2}$$

for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$, then σ is not an SPE with maximum delay n^*.

Notice that lemma 2 directly rules out extreme compatible demand profiles such as $(1,0)$. Fix any set of discount factors and marginal concession costs. A $\hat{z}_2 > 0$ close enough to 0, satisfies inequalities 1 and 2 with $i = 1$. In words, by choosing such a $\hat{z}_2$, player 2 ensures that conditional on 2 choosing S, 1 prefers to concede and get a payoff arbitrarily close to 1 rather than settle for δ_1 or less (inequality 2). By contrast, player 2 has no room to back down since any concession brings a negative payoff (inequality 1). So irrespective of the continuation play, the deviation to $\hat{z}_2$ leads to (A,S) in the concession game, with a positive payoff for player 2. This rules out $(1,0)$ as an equilibrium compatible demand profile.

Elimination of such compatible demand profiles in turn rules out certain continuation plays in equilibrium. Focusing on surplus divisions in SPE with maximum delay n^*, namely B^{n^*}, ensures that in any continuation game the eventually agreed upon division also belongs to B^{n^*}. This recursive structure together with lemma 2 delivers convenient bounds on the corresponding set of equilibrium surplus divisions (the set B^{n^*}).[11]

Proposition 1 *If (z,t) is the outcome of a subgame perfect equilibrium with maximum delay n^*, then*

$$\frac{1-\delta_1}{1-\delta_2^{n^*+1}} \frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1^{n^*+1}}{1-\delta_2} \frac{1+k_2}{k_1}. \tag{3}$$

[11] This step is similar in spirit to the approach taken in Shaked and Sutton (1984) to solve the alternating-offers model.

Proof.

Let $z_i^* = \sup_{z \in B^{n^*}} z_i$. Now suppose for some exactly compatible demand profile z, there exists $\hat{z}_2$ such that

$$\begin{aligned} &1 - z_1 - k_2(z_1 + \hat{z}_2 - 1) < \delta_2^{n^*+1}(1 - z_1^*), \ \textit{and} \\ &1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1) > \delta_1 z_1^*. \end{aligned}$$

Then such a $\hat{z}_2$ also satisfies inequalities 1 and 2 for all $\tilde{z} \in B^{n^*}$ and $1 \leq n \leq n^* + 1$, since for any such $\tilde{z}$ and n it follows that $\delta_2^{n^*+1}(1 - z_1^*) \leq \delta_2^n \tilde{z}_2$ and $\delta_1 z_1^* \geq \delta_1^n \tilde{z}_1$. Therefore, by lemma 2, z cannot arise in any SPE (i.e., $z \notin B^{n^*}$).

Since $z_i^* = \sup_{z \in B^{n^*}} z_i$, there cannot be such a $\hat{z}_2$ for the compatible profile $z = (z_1^*, 1 - z_1^*)$. So there cannot be a $\hat{z}_2 > 1 - z_1^*$ which satisfies both

$$\begin{aligned} &1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \delta_2^{n^*+1}(1 - z_1^*), \qquad \textit{and} \\ &1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1 z_1^*. \end{aligned}$$

These inequalities simplify to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2 - \delta_2^{n^*+1})}{k_2} \qquad \textit{and} \qquad \hat{z}_2 < 1 - \frac{(k_1 + \delta_1) z_1^*}{1 + k_1}.$$

Therefore such a $\hat{z}_2$ cannot exist only if

$$\begin{aligned} &\frac{(1 - z_1^*)(1 + k_2 - \delta_2^{n^*+1})}{k_2} \geq 1 - \frac{(k_1 + \delta_1) z_1^*}{1 + k_1} \\ \Rightarrow &\frac{(1 - z_1^*)(1 - \delta_2^{n^*+1})}{k_2} \geq \frac{z_1^*(1 - \delta_1)}{1 + k_1} \\ \Rightarrow &\frac{1 - \delta_1}{1 - \delta_2^{n^*+1}} \frac{k_2}{1 + k_1} \leq \frac{1 - z_1^*}{z_1^*} \end{aligned}$$

A symmetric argument establishes

$$\begin{aligned} &\frac{1 - \delta_2}{1 - \delta_1^{n^*+1}} \frac{k_1}{1 + k_2} \leq \frac{1 - z_2^*}{z_2^*} \\ \Rightarrow &\frac{z_2^*}{1 - z_2^*} \leq \frac{1 - \delta_1^{n^*+1}}{1 - \delta_2} \frac{1 + k_2}{k_1}. \end{aligned}$$

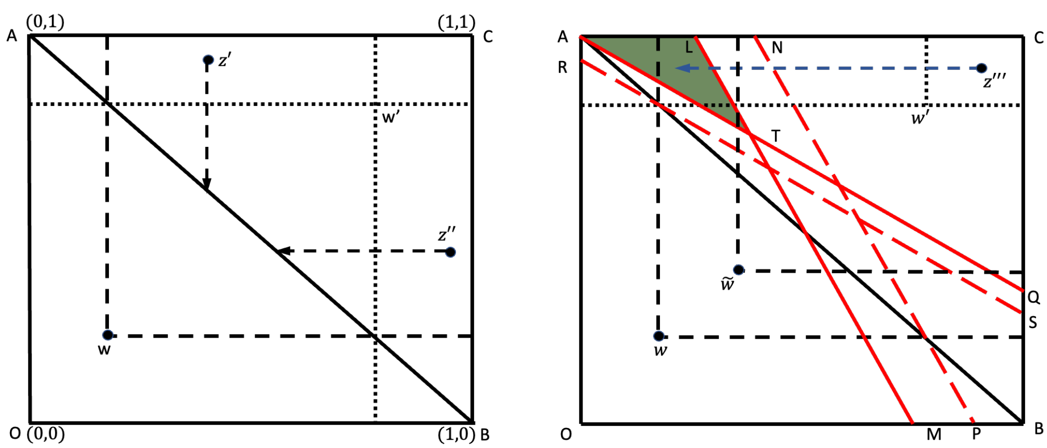
To conclude the proof note that

$$z \in B^{n^*} \Rightarrow \frac{1 - z_1^*}{z_1^*} \leq \frac{z_2}{z_1} \leq \frac{z_2^*}{1 - z_2^*}.$$

■

Proposition 1 describes how limiting the maximum delay permissible in an SPE restricts the range of efficient equilibrium outcomes. I next show that the amount of delay permitted in an SPE is bounded.

Delay requires incompatible demands on the equilibrium path. A bargainer may aim to do better in two ways. Deviate to a compatible demand, or make a milder but still incompatible demand which extracts a concession from the opponent. Requiring such deviations to be unprofitable bounds the amount of delay in SPE. Consider, for instance, the outcome $(z, n+1)$, with delay and present-discounted payoff profile $w = (\delta_1^n z_1, \delta_2^n z_2)$, as depicted in figures 2a and 2b. To be an SPE outcome, the strategy profile must feature n rounds of incompatible demands followed by the exactly compatible demand profile z.



(a) (b)

Figure 2

Figure 2a shows why the period 1 demand profile must lie within the rectangle with w' and C at opposite corners. For all other incompatible demand profiles some player is strictly better off by deviating to a compatible demand. The arrows from demand profiles z' and z'' describe such profitable deviations.

In figure 2b, LM represents the indifference line for player 2 if she faces her best equilibrium continuation payoff.[12] AQ represents the indifference line for player 1 if she faces her worst possible continuation payoff. So any given equilibrium continuation payoff generates indifference lines for players 2 and 1 "above" LM and "below" AQ, respectively, such as RS and NP (from continuation payoff w). Therefore incompatible demands in the dark green region lead to (S, A) *irrespective of equilibrium continuation play.*[13] Recall the strategy profile with delay and payoff profile $w = (\delta_1^n z_1, \delta_2^n z_2)$. Player 1 can deviate from any point in the $w'C$ rectangle to somewhere in the dark green region to the right of the w_1 perpendicular and extract a concession from 2 and obtain a higher payoff, as depicted for the profile z'''. Such a profitable deviation from z''' is not feasible with continuation profile $\tilde{w}$, which involves less delay. This is the feature that bounds the amount of delay in an SPE, as formalized in the following lemma.

Let O^{SPE} be the set of all SPE outcomes and

$$B^* \equiv \{z | (z, t) \in O^{SPE}\}$$

the set of all efficient surplus divisions in such outcomes.

Lemma 3 *Suppose $k_j(k_{-j} - 1) > 1$ for $j \in \{1, 2\}$. If (x, t) is an SPE outcome with $t > 1$, then for $i \in \{1, 2\}$,*

$$\delta_i^{t-1} x_i \geq \frac{1 - \delta_{-i} z_{-i}^*}{1 + k_{-i}} \tag{4}$$

where $z_{-i}^* = \sup_{z \in B^*} z_{-i}$.

Proposition 1 describes how the length of delay permissible in an SPE bounds the set of efficient outcomes that can arise in equilibrium. Lemma 3 captures how the set of efficient equilibrium outcomes limits the maximum delay in an SPE.[14]

[12]Recall from the discussion before figure 1 that the indifference line for player i is given by $1 - z_{-i} - k_i(z_1 + z_2 - 1) = v_i$ where v_i is the present discounted continuation payoff.

[13]For further intuition, recall that the green region in figure 1 changed with changes in the continuation payoff. The dark green region in figure 2b lies inside all such green regions that are feasible given equilibrium continuation payoffs.

[14]The condition $k_j(k_{-j} - 1) > 1$ for $j \in \{1, 2\}$ fails for sufficiently small values of the

The next result combines these two results to obtain a necessary condition for SPE outcomes.

Proposition 2 *Suppose $k_i(k_{-i} - 1) > 1$ for $i \in \{1, 2\}$. If (z, t) is a subgame perfect equilibrium outcome, then*

$$\min\left\{\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1}, \frac{1+k_2}{k_1}(1-\delta_1)\right\} \leq \frac{z_2}{z_1} \leq \max\left\{\frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}, \frac{k_2}{1+k_1}\frac{1}{1-\delta_2}\right\}.$$

Key Implications

Infinite horizon games with simultaneous moves typically yield a folk theorem, in that for discount factors above a threshold *anything goes* under subgame perfection.[15] Proposition 2 shows this to be false for the current model. Further, unlike in the closely related IH-NDG, the maximum delay in SPE is bounded.

More importantly, a unilateral increase in the concession cost function for a bargainer generates better equilibrium outcomes for her. This confirms Schelling's insight about weakness being a strength. Greater patience is similarly beneficial. This preserves a key implication of the canonical bargaining models. Both these features are easiest to observe in Proposition 1. From the latter, it also follows that a bilateral increase in marginal concession costs, holding its ratio fixed, narrows down the set of equilibrium predictions.[16]

Proposition 1 also delivers a simple way to classify possible equilibrium surplus divisions on the basis of the maximum anticipated delay following any history. These are directly testable in laboratory and field experiments that collect information about bargainers' beliefs about delay in addition to standard outcome data. Finally, it also follows that additional conditions that restrict the maximum delay allowed in an SPE, as a result, also shrink the set of compatible demand profiles that can obtain in equilibrium. I study three such conditions in the following subsections.

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marginal concession costs. Qualitatively, concession costs are less salient in the model at such parameter values, and therefore impose less constraints on the length of delay that can arise in SPE.

[15] More precisely, any payoff profile that is feasible and individually rational can be achieved in SPE.

[16] I return to this feature in more detail in sections 3.2 and 4.

3.2 Renegotiation-Proofness

Negotiators who are familiar with each other should, in the presence of multiple equilibria, be able to avoid the strictly Pareto dominated ones. This is especially so, if the Pareto dominating equilibrium is one they anticipate to play following some history. Since the game is identical following any history $h \in H$, the negotiators would see the incongruence of taking an efficient path following one such history and an inefficient one following another. Given their familiarity they need not take their cues from some possibly inefficient norm, but rather count on renegotiating away from such inefficient equilibria. The notions of *weak renegotiation proofness* in Farrell and Maskin (1989) and *internal consistency* in Bernheim and Ray (1989) capture this idea in the context of repeated games. While not a repeated game, the present model shares its key feature that following any number of rounds (of failed bargaining), the continuation game looks the same. Relying on this stationarity, I import an appropriate notion of renegotiation-proofness for the current setting.

Let $\psi(\sigma; h^t)$ be the continuation payoff (profile) implied by σ given history $h^t \in H$ and let

$$\Psi(\sigma) = \cup_{h^t \in H} \psi(\sigma; h^t)$$

be the set of all continuation payoffs under σ.

Definition 2 *An SPE σ is renegotiation-proof if for no $x, y \in \Psi(\sigma)$ is $x \gg y$.*

Renegotiation-proofness is routinely studied in equilibrium analyses for a variety of economic questions, and with important implications. See, for instance, Barrett (1994) on international environmental agreements, Matsuyama (1990) on trade liberalization, Kletzer and Wright (2000) on sovereign debt. Nonetheless, it is not the only "reasonable" description of behaviour. It does however facilitate a natural separation of all pairs of bargainers into those who always coordinate on efficient outcomes on the equilibrium path and anticipate the same off it, and others. Renegotiation-proofness delivers a sharper characterization of the behaviour of the former group.

Note that renegotiation-proofness does not rule out history dependent strategies. Consider, for instance, the construction due to Avner Shaked reported in

Sutton (1986). It supports any efficient division of the surplus as an SPE outcome of a 3-person Rubinstein bargaining game for high enough discount factors. The construction relies heavily on the history-dependence of the strategy profile. Imposing an appropriate version of renegotiation-proofness has no effect on the result since all continuation outcomes are efficient. The severe multiplicity persists. In the current model, however, renegotiation-proofness sharply restricts the set of equilibrium outcomes.

Proposition 3 (z, t) *is the outcome of a renegotiation-proof subgame perfect equilibrium if and only if* $t = 1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \le \frac{z_2}{z_1} \le \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}. \tag{5}$$

I now sketch the argument behind this result. The detailed proof is in the appendix. Given the structure of SPE identified in lemma 1, renegotiation-proofness simply rules out any delay. The necessity of inequality 5 then follows immediately from proposition 1. To establish sufficiency, I construct the following stationary strategy profile, which I show to be subgame perfect for any z satisfying inequality 5 in lemma 5 in the appendix.

Construction 1 *Consider the following stationary strategy profile,* σ*. Fix* z *such that* $d(z) = 0$*. For all* $h^t \in H$*, set* $\sigma_i(h^t) = z_i$*. If player* i*, for some* $i \in \{1, 2\}$*, in period* t *deviates to a higher demand,* $\hat{z}_i > z_i$*, then in the concession stage game* (S, S) *is played if it is a Nash equilibrium and otherwise* (A_i, S_{-i}) *is played. For all other* $h \in H'$ *some pure strategy Nash equilibrium of the concession stage game is played.*

The strategy profile σ above satisfies renegotiation-proofness, since following any history $h \in H$ the continuation outcome is efficient and consists of agreeing on the compatible demand profile z.

Proposition 3 offers a preview of the limit uniqueness result in section 4.2. Consider a sequence of these linear bargaining games parametrized by marginal concession costs $\{k_1^n, k_2^n\}_{n=1}^{\infty}$ such that $k_1^n = \gamma k_2^n$ for all n and $k_2^n \to \infty$ as $n \to \infty$. Observe first that at the limit, it is too costly for any bargainer to concede following any incompatible demand. The model therefore reduces to the IH-NDG. However, in contrast to the acute multiplicity of SPE in the IH-NDG, the set of renegotiation-proof SPE as characterized in proposition 3 converges to a singleton at the limit. At this unique limit outcome, the bargainers agree on the compatible profile z with

$$\frac{z_2}{z_1} = \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma}.$$

Collard-Wexler et al. (2019) use a criteria called *no-delay* to refine the set of SPE in their bargaining model. The criteria is identical to the notion of *SPE with maximum delay 0*, in that it requires no delay following any history. All results requiring renegotiation proofness in this paper would remain unchanged if the no-delay criteria was used instead. This however is a result that follows from proposition 3. In the very closely related IH-NDG, by contrast, perpetual disagreement satisfies renegotiation proofness while obviously violating no-delay.

3.3 Markov Perfect Equilibria

Negotiations often take place between strangers or relatively inexperienced bargainers. The assumption of renegotiation-proofness may be inappropriate in such cases. A different assumption, routinely made in applied work, requires players to use Markov strategies. Maskin and Tirole (2001) discusses some of the theoretical considerations that support its use. Vespa (2020) finds experimental evidence of it as the modal behaviour in the dynamic common pool game. In this section I focus on SPE in Markov strategies. Similar to the previous section, the agenda is not to propose the Markov restriction as the only "sensible" one. Instead, it is to obtain a sharper characterization of the behaviour of an empirically large and relevant group of people in this setting.[17]

Definition 3 *σ_i is a Markov strategy for player i if for all $h, \tilde{h} \in H^t$*
(i) $\sigma_i(h) = \sigma_i(\tilde{h})$ and
(ii) $\sigma_i(h, z^{t+1}) = \sigma_i(\tilde{h}, z^{t+1})$.

[17] Note that unlike in the repeated prisoner's dilemma, Markov strategies do not preclude efficiency in the current bargaining game.

In words, under the Markov requirement, player i's demand in period t must be invariant to the specific $t-1$ demand profiles rejected in the past. Further, the concession stage decision in period t should depend upon the period t demand profile alone. Note, however, that it allows demands and concession stage behaviour to depend on calendar time. For instance, a strategy in which the demands get less and less extreme over the first m periods of bargaining is permitted. Indeed, such strategies can generate delay in equilibrium.

Define n^{MB} as the largest $n \in \mathbb{N}$ that satisfies,

$$\frac{k_1}{\delta_1^n} + \frac{k_2}{\delta_2^n} \leq k_1 + k_2 + k_1 k_2.$$

Let O^{AMP} be the set of outcomes, such that $(z, t+1) \in O^{AMP}$ implies

$$t \leq n^{MB} \qquad \textit{and} \qquad \frac{1-\delta_1^t}{\delta_2^t(1+k_1)-1} \leq \frac{z_2}{z_1} \leq \frac{\delta_1^t(1+k_2)-1}{1-\delta_2^t}.$$

Proposition 4 *(a) If $(z, t+1)$ is an MPE outcome then $(z, t+1) \in O^{AMP}$ and*

$$\frac{k_2}{1+k_1} \min\left\{\frac{1-\delta_1}{1-\delta_2}, \frac{1-\delta_1^{n^{MB}+1}}{1-\delta_2^{n^{MB}+1}}\right\} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1} \max\left\{\frac{1-\delta_1}{1-\delta_2}, \frac{1-\delta_1^{n^{MB}+1}}{1-\delta_2^{n^{MB}+1}}\right\}. \tag{6}$$

(b) If $(z, t+1) \in O^{AMP}$ satisfies

$$\frac{k_2}{1+k_1}\frac{1-\delta_1^n}{1-\delta_2^n} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1}\frac{1-\delta_1^n}{1-\delta_2^n} \tag{7}$$

for some $1 \leq n \leq t+1$, then $(z, t+1)$ is an MPE outcome.

The detailed proof is in the appendix. I sketch the argument here. As with SPE, to sustain delay in MPE the bargainers must make incompatible demands that neither wish to deviate from. To ensure that a unilateral deviation to a compatible profile is not profitable, the demands simply need to be sufficiently aggressive, exactly as in the case of SPE. It is less demanding to rule out profitable deviations where a bargainer makes a lower but still incompatible demand and extracts a concession from her opponent. Long anticipated delay makes such deviations feasible since it lowers the payoff from disagreement and makes concession more palatable. Due to the Markov restriction, the continuation play cannot change following such a deviation. This feature further limits the amount of delay that can arise in an MPE as well as the range of eventual surplus divisions. These constraints characterize the set of outcomes O^{AMP}.

The recursive structure of the game and MPE is used next to characterize the best compatible demand profile that can arise for each player. Equilibrium continuation plays are allowed to involve any length of delay within the bound identified above. This delivers the necessary condition. The sufficiency condition uses a stronger recursive structure. Any compatible demand profile announced by the bargainers is followed by a continuation play that features the same length of delay n.

The final result in this section characterizes the set of stationary MPE outcomes.[18] Stationarity does not allow strategies to depend on calendar time. It requires

$$\sigma_i(h) = \sigma_i(\tilde{h}) \qquad \forall h, \tilde{h} \in H.$$

Proposition 5 (z, t) *is a stationary Markov perfect equilibrium outcome if and only if* $t = 1$ *and*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

Key Implications

The set of stationary MPE outcomes coincides exactly with the set of renegotiation-proof SPE outcomes. Typically, in dynamic games where both concepts apply, stationary Markov perfection is strictly more restrictive than renegotiation proofness.[19] Effectively, renegotiation-proofness rules out non-stationary strategic behaviour of a specific kind, one that leads to multiple outcomes that can

[18]The proof is in the appendix but follows immediately from the observation that a stationary MPE features either immediate agreement or perpetual delay and the latter can be ruled out. The result then follows from lemma 5 and proposition 1.

[19]Take an infinitely repeated prisoner's dilemma game with high enough discount factors, for instance. The unique MPE outcome involves both parties defecting forever. On the other hand, cooperation can be sustained as a weak renegotiation-proof SPE, as shown in Farrell and Maskin (1989) and van Damme (1989).

be Pareto ranked. Stationary MPE typically does more by ruling out all non-stationary strategic behaviour. In the current model, every outcome that survives renegotiation-proofness can be supported by stationary strategies, which robs stationary Markov perfection of any additional bite.

Markov perfection without the added constraint of stationarity, however, does not imply renegotiation-proofness. Since continuation play can vary by calendar time, two subgames which "look the same" may feature equilibrium outcomes that can be Pareto ranked. This is precisely what happens in the current model, wherein bargainers make a series of incompatible demands followed by an agreement.[20] Proposition 4 has two key implications in this setting.

First, if the bargainers are equally impatient then the set of equilibrium compatible profiles in an MPE coincides with that in renegotiation-proof SPE, even though the latter does not feature delay while MPE may. The set consists of all exactly compatible demand profiles z such that

$$\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1}.$$

Second, with unequal impatience the set of equilibrium compatible profiles is larger under MPE than under renegotiation-proofness. Remarkably, the best compatible profile for the more patient player remains the same under both specifications, while the less patient player may do better. For instance, inequality 6 translates to

$$\frac{k_2}{1+k_1}\frac{1-\delta_1}{1-\delta_2} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1}\frac{1-\delta_1^{n^{MB}+1}}{1-\delta_2^{n^{MB}+1}},$$

when $\delta_1 \geq \delta_2$.[21]

For the intuition behind these findings return to figure 1. It shows how the best equilibrium compatible profile for player 1 (label it z^*) given the present-discounted continuation profile w , marked by the point L, is pinned down by the intersection of the two indifference lines. The continuation outcome has

[20] In an equilibrium with delay, the overall game and the subgame starting in the period where agreement is reached "look the same" and yet feature different outcomes that can be Pareto ranked.

[21] Since $\delta_1 \geq \delta_2 \Leftrightarrow \frac{1-\delta_1^t}{1-\delta_2^t} \leq \frac{1-\delta_1^{t+1}}{1-\delta_2^{t+1}}$, as shown in Lemma 6 in the appendix.

two components, the (eventual) compatible profile, say $\tilde{z}$, and the delay to that agreement, say t. Two key properties obtain. First, z_1^* is increasing in $\tilde{z}_1$ and second, $\frac{\partial z_1^*}{\partial \tilde{z}_1} < 1$.

Under Markov perfection, the continuation outcome $(\tilde{z}, t+1)$ that yields the best equilibrium profile for player 1 may feature delay but no greater than n^{MB}, and $\tilde{z}$ itself must arise in some MPE. Restricting attention to MPEs where any compatible demand is followed by continuation play with the same length of delay t, permits a simple characterization of the best equilibrium profile for player 1 (in this restricted set of MPEs), say z^{t*}, as the solution to a fixed point problem. For each delay t between 0 and n^{MB} the corresponding best profile for player 1 satisfies the equation $\frac{1-z_1^{t*}}{z_1^{t*}} = \frac{k_2}{1+k_1}\frac{1-\delta_1^{t+1}}{1-\delta_2^{t+1}}$. The two key properties highlighted earlier then ensures that the best (unrestricted) MPE compatible profile for player 1, z^*, must satisfy the inequality

$$\min_{0 \leq t \leq n^{MB}} \frac{k_2}{1+k_1}\frac{1-\delta_1^{t+1}}{1-\delta_2^{t+1}} \leq \frac{1-z_1^*}{z_1^*}.$$

The bounds identified above, one for each permissible length of delay, all coincide in the case of equal impatience, which leads to the first key implication of Proposition 4. In the case of unequal impatience, if player 1 is the more patient one, then she can never do better in an MPE with delay than one without. Indeed with $\delta_1 > \delta_2$, the bound above is attained at $t = 0$.

4 General Model

The qualitative results obtained in propositions 1 and 3 do not rely on the assumption of linearity. Let $\mathcal{U}$ and $\mathcal{C}$ be the set of all (pairs of) functions that satisfy assumptions 1 and 2, respectively. Fix some $u \in \mathcal{U}$ and $c \in \mathcal{C}$ and $n^* \in \mathbb{N}$. For $i \in \{1,2\}$, define the function $\tilde{z}_{-i}^{n^*}(z_i)$ implicitly as the solution to the equation

$$u_{-i}(1-z_i) - c_{-i}(z_i + \tilde{z}_{-i}^{n^*}(z_i) - 1) = \delta_{-i}^{n^*+1} u_{-i}(1-z_i). \tag{8}$$

Similarly define the function $\tilde{\tilde{z}}^{n^*}_{-i}(z_i)$ implicitly as the solution to the equation

$$u_i(1-\tilde{\tilde{z}}^{n^*}_{-i}(z_i)) - c_i(z_i + \tilde{\tilde{z}}^{n^*}_{-i}(z_i) - 1) = \delta_i u_i(z_i). \tag{9}$$

It turns out that there is a unique z_i, which I denote as $z_i^{Mn^*}$, that solves

$$\tilde{\tilde{z}}^{n^*}_{-i}(z_i) = \tilde{z}^{n^*}_{-i}(z_i). \tag{10}$$

Proposition 6 *In the general model, if (y,t) is the outcome of an SPE with maximum delay n^*, with $y = u(z)$, then $d(z) = 0$ and*

$$\frac{1-z_1^{Mn^*}}{z_1^{Mn^*}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{Mn^*}}{1-z_2^{Mn^*}}. \tag{11}$$

Relabel $z_i^{Mn^*}$ simply as z_i^M when $n^* = 0$, and the following generalization of Proposition 3 obtains.

Proposition 7 *In the general model, (y,t) is the outcome of a renegotiation-proof SPE with $y = u(z)$, if and only if $t = 1$, $d(z) = 0$ and*

$$\frac{1-z_1^{M}}{z_1^{M}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{M}}{1-z_2^{M}}. \tag{12}$$

Figure 3 clarifies the content of these results. For a given n^*, equations 8 and 9 generate four functions. The function $\tilde{z}_2^{n*}(z_1)$ returns the smallest player 2 demand such that following any pair of incompatible demands z_1 and $z_2 > \tilde{z}_2^{n*}(z_1)$ and a continuation payoff of $\delta_2^{n^*+1}(1-z_1)$, player 2 strictly prefers S. The function $\tilde{\tilde{z}}_2^{n*}$ returns the largest player 2 demand such that following any pair of incompatible demands z_1 and $z_2 < \tilde{\tilde{z}}_2^{n*}(z_1)$ and a continuation payoff $\delta_1 u_1(z_1)$, player 1 strictly prefers concession, if 2 chooses S.

Notice that $z_1 > z_1^{Mn^*}$ implies $\tilde{\tilde{z}}_2^{n*}(z_1) > \tilde{z}_2^{n*}(z_1)$. Such a z_1 cannot be the best equilibrium efficient split for player 1, because player 2 could then deviate to a demand $z_2 \in (\tilde{z}_2^{n*}(z_1), \tilde{\tilde{z}}_2^{n*}(z_1))$. This would force a concession from player 1 in the resulting concession game, *for any equilibrium continuation payoff*. This is the argument behind the bound $z_1^{Mn^*}$. A symmetric argument applies to player 2's bound of $z_2^{Mn^*}$.

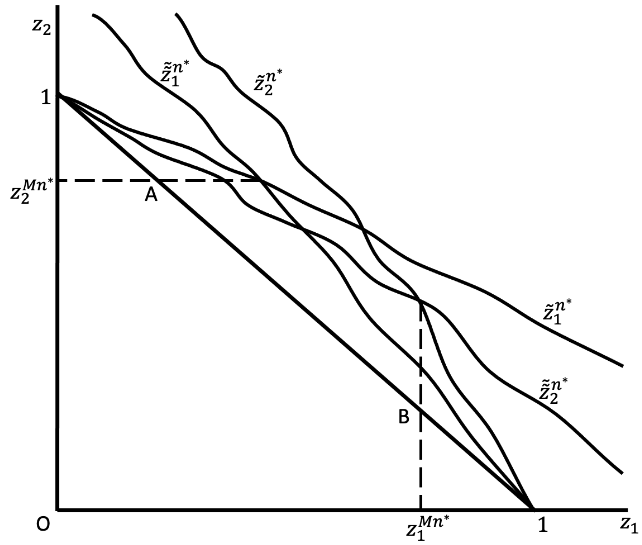

Figure 3

Increasing player 1's cost function, holding all else fixed, leaves functions $\tilde{\tilde{z}}_1^{n*}$ and $\tilde{z}_2^{n*}$ unchanged while moving $\tilde{\tilde{z}}_2^{n*}$ and $\tilde{z}_1^{n*}$ closer to the efficient frontier. Figure 3 shows that this would increase $z_1^{Mn^*}$ and lower $z_2^{Mn^*}$, confirming Schelling's insight about weakness being a strength in this more general setting. Increasing player 1's patience, δ_1, has a qualitatively similar effect to increasing her cost function. Finally observe that lowering n^*, leaves $\tilde{\tilde{z}}_i^{n*}$ unchanged for $i \in \{1, 2\}$ while moving both $\tilde{z}_1^{n*}$ and $\tilde{z}_2^{n*}$ closer towards the diagonal. This means that $z_i^{Mn^*}$ is increasing in n^*, as expected.

4.1 Kalai Bargaining Solution

Kalai (1977) introduces a family of bargaining solutions parametrized by a single variable, a proportion. Any bargaining solution that is monotonic, in that increasing the set of feasible bargaining outcomes never hurts either bargainer (formally defined below), is a Kalai (or proportional) bargaining solution (KBS) and vice versa. The family of solutions is exactly characterized by the axioms of independence of irrelevant alternatives, individual monotonicity and continuity. In addition to being compelling theoretically, the solutions are used extensively and in a variety of fields. Recently, for instance, it is used increasingly in the field of monetary economics.[22]

I now introduce some notation in order to define KBS. Let $\Pi(u) = \{y | y_i = u_i(z_i), z_i \geq 0, \forall i \in \{1,2\} \textit{ and } z_1 + z_2 \leq 1\}$ denote the set of feasible payoffs that can arise from some allocation of the surplus. Set $u^d = (u_1(0), u_2(0)) = (0,0)$ to be the disagreement point. Combined, $(\Pi(u), u^d)$ represents a bargaining problem. Finally let $\mathcal{B} = \{(\Pi(u), u^d) | u \in \mathcal{U}\}$ be the set of all bargaining problems that can arise from payoff functions that satisfy assumption 1. A bargaining solution is a function $\phi : \mathcal{B} \rightarrow \mathbb{R}^2$ such that $\phi(B) \in B$ for all $B \in \mathcal{B}$. It is monotonic if for any $A, B \in \mathcal{B}$, $A \subset B$ implies $\phi(B) \geq \phi(A)$.

The Kalai Bargaining Solution with proportions $(\theta, 1)$, denoted by $\mathcal{K}_\theta$, is defined as

$$\mathcal{K}_\theta(\Pi, u^d) = \lambda(\Pi, u^d) \cdot (\theta, 1), \forall \Pi \in \mathcal{B}$$

where $\lambda(\Pi, u^d) = \max\{q \in \mathbb{R} | q \cdot (\theta, 1) \in \Pi\}$.[23] In words, the proportion parameter, θ, fixes a unique ray in the utility space passing through $(0,0)$. For any bargaining problem, the KBS with proportion θ then simply picks the point where the ray meets the efficient frontier of the bargaining problem.

4.2 Strategic Foundation

Return now to the general non-cooperative bargaining model. Making the concession cost functions steeper makes it progressively harder for the bargainers to back down from their demands. At the limit, with arbitrarily high marginal concession costs, the infinite horizon version of the Nash demand game obtains. Neither player can back down from incompatible demands. Binmore (1987) points out that any efficient payoff profile can be supported as an SPE outcome of the IH-NDG. Infinite delay can also be supported in SPE by each bargainer always demanding the entire surplus. Chatterjee and Samuelson (1990) show that this acute multiplicity further survives trembling hand perfection (see Selten (1975)). The limit set of renegotiation-proof SPE outcomes, in sharp contrast, is a singleton.

[22] See, for instance, Lagos, Rocheteau and Wright (2017). Duffy, Lebeau and Puzzello (2021) find that KBS better fits the behaviour of bargainers in the laboratory facing liquidity constraints.

[23] Note that $(\theta, 1)$ is a vector and since q is a scalar, $q \cdot (\theta, 1) = (q\theta, q)$.

For any $u \in \mathcal{U}$ and $c \in \mathcal{C}$, let $g^c(u)$ denote the game described in section 2, where u_i and c_i are player i's payoff and concession cost functions, respectively, for $i \in \{1,2\}$. Denote the corresponding set of renegotiation-proof SPE payoff profiles by $\xi(g^c(u))$. g^c therefore maps any pair of payoff functions in $\mathcal{U}$ to its corresponding infinite horizon bargaining game. Consider a sequence of such mappings $\{g^{c^n}\}_{n=1}^{\infty}$ with $c^n \in \mathcal{C}$ for all n, such that as $n \to \infty$, $c_i^{n\prime}(0+) \to \infty$ (the right derivative of the concession cost functions at 0 becomes arbitrarily large). Next, assume that there exists some integer N such that $\forall m, n > N$,

$$0 < \lim_{d \searrow 0} c_1^m(d)/c_2^m(d) = \lim_{d \searrow 0} c_1^n(d)/c_2^n(d) < \infty. \tag{13}$$

In words, the assumption requires that sufficiently far along the sequence, the ratio of the concession costs for vanishingly small concessions is the same, positive and bounded. The assumption is satisfied by the linear specification in section 3, but it does not require linearity of either the individual cost functions or even their ratio.[24]

Finally let

$$\xi_\gamma^*(u) = \lim_{n \to \infty} \xi(g^{c^n}(u)), \qquad \textit{where } \gamma = \lim_{n \to \infty} c_1^n(0+)/c_2^n(0+).$$

The limit set of renegotiation-proof SPE is therefore captured by $\xi_\gamma^*(u)$. It is parameterized by γ, which is the ratio of the concession cost functions evaluated at the limit as the concessions become vanishingly small. The assumption described in 13 ensures that γ is well defined.

Proposition 8 *For all $u \in \mathcal{U}$, $\xi_\gamma^*(u) = \mathcal{K}_\theta(\Pi(u), u^d)$ where $\theta = \gamma(1-\delta_2)/(1-\delta_1)$.*

Intuition

Nash equilibria of non-cooperative games are invariant to affine transformations of agents' utility functions while the Kalai solution is *not*. This creates an important obstacle that any strategic foundation for the Kalai solution must overcome.[25] It is achieved in Proposition 8 by obtaining the Kalai solution as

[24] Consider, for example, $c_1^n(d) = n(d + 2d^2)$ and $c_2^n(d) = n(4d + d^2)$.

[25] I thank an anonymous referee for emphasizing this feature.

the limit equilibrium outcome of a particular sequence of games. The utility function of agent i in any game along this sequence, say n, is itself a function of both the payoff function u_i and the cost function c_i^n (in an additively separable way). Therefore the set of RP-SPE outcomes for the game $g^{c^n}(u)$, denoted by $\xi(g^{c^n}(u))$ is *not* invariant to affine transformations of u alone (holding c^n fixed), and likewise neither is its limit, $\xi^*_\gamma(u)$.

Recall that a Kalai solution (i.e. with a particular proportion) maps any pair of utility functions, one for each bargainer, to an outcome on the Pareto frontier with the property that the ratio of the utilities corresponds to the fixed proportion. The strategic foundation in this paper similarly takes any pair of utility functions, one for each bargainer, and models it as the payoff functions in a strategic game which has the additional ingredients of discount factors and concession cost functions. It then finds the limit equilibrium payoff profile of the game as the concession cost functions are made arbitrarily steep. The limit equilibrium payoff profile admits a characterization where the ratio of the utility to the two bargainers is equal to a constant which is independent of the utility functions themselves. This is precisely why it coincides with a Kalai solution whose proportion aligns with the constant.

The constant identified in the strategic foundation is a function of the bargainers' discount factors and a limiting ratio of the concession cost functions. The latter is in a sense a measure of relative concession costs evaluated at a limit where the concession cost functions are arbitrarily steep.

The intuition for the characterization result can be split into three key arguments. First, for any set of cost and payoff functions, the two distinct best renegotiation proof SPE outcomes (henceforth labeled extreme RP-SPE outcomes), one for each player, admit a stationary strategy profile each. This equilibrium feature of the current model does not hold for general dynamic bargaining games.[26] It then follows that an agent's best renegotiation proof SPE outcome is identical to her best stationary SPE outcome.

Second, the two extreme RP-SPE outcomes approach each other as the marginal concession cost increases. This can be seen in figure 3, setting $n^* = 0$.

[26] For instance, the extreme RP-SPE outcomes in the 3-person Rubinstein bargaining game reported in Sutton (1986) do not admit a stationary strategy profile.

The extreme RP-SPE outcomes are $(z_1^M, 1 - z_1^M)$ and $(1 - z_2^M, z_2^M)$. As the marginal concession cost increases, all the functions $\tilde{z}_i^0$ and $\tilde{\tilde{z}}_i^0$ for $i \in \{1, 2\}$, swing towards the efficient demand frontier. In turn, the two extreme RP-SPE outcomes and, as a result all RP-SPE outcomes, converge. Very importantly, such convergence does not occur for the two best SPE outcomes in general. Higher marginal concession costs permit longer equilibrium delay, which can be used to construct equilibria that keep the two outcomes apart.

Finally, at the limit the ratio of payoffs to the two bargainers equals a constant. The properties of this constant are central to the strategic foundation result. First, the constant is independent of the payoff functions, and as stated above, this is exactly why the limit outcome aligns with a Kalai solution. Second, the value of the constant pins down the relevant proportion of the Kalai solution. Lastly, the constant is a function solely of the discount factors and a limit ratio of the concession cost functions and therefore these parameters endogenously determine the proportion of the Kalai solution. These properties of the limit characterization rely on the additive separability of the concession costs and their dependence on the distance between "physical" demands instead of payoff levels.

The strategic foundation for KBS in Dutta (2012), which studies the single period version of the current model, is similarly obtained at the high concession cost limit. The convergence argument in that paper is substituted by the second and third arguments above, to fit the infinite horizon environment and the requirements of subgame perfection.

High concession cost limit

The high concession cost setting is best interpreted as a perturbation of the perfect commitment implicit in the IH-NDG. In the latter, agents are fully committed to their demands in that incompatible demands directly lead to an impasse in that period. The current analysis shows that allowing agents even the smallest room for concession opens up strategic considerations that substantially shrink the set of equilibrium outcomes. That perturbing the commitment structure selects the KBS, stands in sharp contrast to other equilibrium selection arguments for the NDG that deliver the Nash bargaining solution (see Nash (1953), Binmore (1987a), Carlsson (1991)) . These latter arguments perturb the information structure in a way that effectively smooths the NDG payoff function, in that the payoff following incompatible demands smoothly tapers off to zero as a function of the demands. The IH-NDG with a similar information structure perturbation is obtained as a special case of a more general bargaining model studied in Harstad (2023). Here too, the argument selects the asymmetric Nash bargaining solution. The commitment structure perturbation in the current paper does not smooth the (effective) payoff functions. They remain discontinuous at the efficient demand frontier.

The Nash program

A narrow reading of the Nash program simply calls for strategic games whose equilibria align with a given cooperative solution concept. A broader interpretation requires further that the strategic models capture key features of some class of institutional frameworks.[27] This facilitates the use of the corresponding bargaining solution for negotiations that take place in such institutional settings. The preceding analysis makes such a case for the Kalai solution in environments where commitment ability by way of concession costs is prevalent. Section 5.1 lists some such institutional and social environments.

It is clear in Kalai (1977) that while the family of bargaining solutions is a compelling one, finding the relevant proportion needs information beyond what is modelled in a standard bargaining problem (an element of $\mathcal{B}$). In proposition 8 the degree of impatience of the bargainers and their relative concession costs constitute this information. So not only does this formalization of Schelling's theory provide a strategic foundation for the Kalai bargaining solution, it also selects the appropriate proportion.

Implications

The characterization in proposition 8 is easy to interpret. The physical split of surplus must be efficient ($z_1 + z_2 = 1$) and the resulting ratio of utilities must satisfy

$$\frac{u_1(z_1)}{u_2(z_2)} = \gamma \frac{1-\delta_2}{1-\delta_1}.$$

[27]See quoted passage from Binmore, Osborne and Rubinstein (1992) in the introduction.

Greater patience leads to higher payoff, as in canonical models of bargaining. In essence, despite its definition as a limit, γ captures the relative concession costs faced by the players. Higher concession costs translate into better bargaining outcomes. The limit solution, like the KBS it is equivalent to, satisfies monotonicity and fails scale invariance. An important implication is that scaling up a specific agent's payoff function brings that agent a higher payoff (as required by monotonicity) but also a lower physical split of the surplus. Consider the following example, borrowed from Kalai (1977) that involves two scenarios of splitting one hundred chips. In both, the bargainers have the same linear utility for money, the same discount factors and $\gamma = 1$. In the first scenario either player can cash in each chip for 1 dollar. In the second, player 2 can continue to cash in each chip for one dollar while player 1 can cash in each chip for three dollars. In the limit solution the players get 50 dollars each from a 50-50 split in the first scenario and 75 dollars each from a 25-75 split in the second scenario. The Nash bargaining solution, by contrast, calls for a 50-50 split in both scenarios. Note that the difference arises even in a wholly linear specification.

5 Discussion

5.1 Concession Costs

The commitment ability at the heart of the bargaining model is generated by the cost an agent must pay from backing down from her current incompatible demand.[28] It is therefore important to discuss the relevant features of these concession cost functions. Following the Nash program mandate, I begin by listing a few ways in which such costs arise and the forms they take.

Examples

(i) *Audience Costs*: Elected representatives negotiating on behalf of their constituents are punished with a dimmer re-election prospect (the cost) for backing down from a publicly announced demand. In international negotiations this cost is generated by the domestic political audience, and has been studied in some detail following the work of Fearon (1994) and Martin (1993). Tomz (2007) pro-

[28]By contrast, bargainers are not held to historical demands that led to an impasse.

vides direct evidence of these audience costs through experiments embedded in public opinion surveys.

In domestic negotiations between rival political parties the level of public support generated by each competing demand determines its audience cost. The greater the support for an announced demand the higher the cost of backing away from it. Leventoglu and Tarar (2005) and Basak and Deb (2020) study such concession costs in a Rubinstein bargaining model, where each player gets only one attempt at commitment.

(ii) *Delegated Bargaining*: Negotiations between two entities are often carried out by representatives (delegates) armed with appropriate incentives. A penalty for backing down from an announced demand is one such incentive. Indeed, the example in Schelling (1956) of a union official bargaining on behalf of the members has this feature; concession raises the odds of the official getting fired. Under this interpretation of concession costs, the form of delegation is exogenous in this paper. For other forms of (endogenous) delegation in bargaining see Crawford and Varian (1979), Jones (1989), Segendorff (1998) and Harstad (2008).

(iii) *Face*: Perhaps the most pervasive form of concession costs, but the least studied in economics, consists of *losing face*. Carefully detailed in Ho (1976), the concept of face, Chinese in origin, corresponds to a notion of social standing that is distinct from status, prestige, dignity, and the like. Unlike a binary variable, it can vary quantitatively in a gradual manner. Furthermore, the relevant quantity of face for an individual depends on the social situation of the interaction. As Ho points out, *It is the extent to which a particular person's social functioning is adversely affected that constitutes the true measure of what losing face means to him.* In the current setting, concession leads to losing face. This form of concession costs allows for a variety of social, political and historical features to translate into bargaining power in the model. For example, concession may lead to greater (or lesser) loss of face for a man compared to a woman, depending on gender norms.

Concession

In the model, backing down from a stated demand in a given round of bargaining and accepting the lower competing offer incurs a concession cost. On the other hand, following an incompatible demand that led to an impasse, a lower demand in a subsequent round of bargaining carries no such penalty. The assumption here is that the explicit concession within an active round of bargaining in the first scenario is evaluated differently (by the political audience, society or self, depending on the source of the concession cost) from the implicit concession in the second. This may happen for a number of reasons. For instance, the reaction of a domestic political audience to the evolution of an international negotiation may be driven by the latest news cycle, which would typically have greater coverage of any explicit concession in an active round of bargaining. Further, demands in negotiations tend to be more involved descriptively than mere points on the unit interval, as we typically assume in our theoretical models. As a result, it may be easier to recognize an instance of explicit concession as compared to an implicit one.[29]

The modelling assumption has two implications. First, the resulting commitment ability is short lived and second, bargainers can make multiple attempts at commitment. Leventoglu and Tarar (2005) and Basak and Deb (2020) make the opposite assumption, wherein bargainers attempt to commit only once with an initial round of announced demands. They then engage in Rubinstein bargaining with the feature that any eventually agreed upon share, if less than the initial demand, carries a concession cost. An interesting implication of these two approaches to modelling commitment is the limit model as the concession costs are made arbitrarily high. At the limit, these models with a single opportunity to commit lead to the static NDG while the current model leads to the IH-NDG.

[29] Kirky (1995) gives an account of the negotiations from 1985 to 1988 between the United States and Canada regarding claims over the waters of the Canadian Arctic archipelago. At an early round, the publicly stated Canadian claim to sovereignty concerning the waters of the archipelago, which led the Canadian public to perceive the issue as a challenge to its sovereignty, allowed the Canadian side little room for compromise. However, they did not face a similar constraint when they sidestepped the sovereignty issue in a later round, which in effect was a climb down from their earlier demand.

Structure of the cost function

The qualitative results of the bargaining model, as in section 4, require only that the concession cost functions satisfy assumption 2. This is consistent with the examples above that suggest little structure for the functions other than it be increasing in the conceded amount. In particular, the cost functions need not be linear (as in section 3) or even convex. In many models of economic decision making, the cost from taking some productive action is assumed to be convex to ensure the overall objective function remains concave and admits an interior optimal solution. In the current setting, concession costs are incurred only off the equilibrium path. Therefore, despite their key role in the model, the curvature of these functions plays no role.

In line with the examples above, an agent's concession cost in the model is, in a sense, independent of how much she cares about the surplus. For instance, the audience cost faced by an elected representative is determined by how much the domestic audience cares to punish the agent for different degrees of concession. The cost is not directly related to how much the agent herself values different surplus splits. This feature is captured in two ways by the model. First, the size of the concession depends on the agents' "physical" demands and not payoff levels. Second, the concession cost is additively separable. These in turn make the (large marginal cost) limit ratio of the commitment costs independent of the payoff functions, which is a key component of proposition 8. Like the Kalai solution, the equilibrium prediction at this limit varies with the payoff function.

Relation to renegotiation proofness

The choice to concede is an individual one (made in equilibrium) and arises *after* incompatible demands. It depends on how far apart the incompatible demands are, the cost function and anticipated future play. Renegotiation-proofness rules out incompatible demands *before* they are made, by requiring agents to coordinate away from Pareto dominated equilibria. Once incompatible demands are made, however, concession (in that period) is costly. Higher concession cost functions reflect sharper incentives from sources like in the examples above, and are unrelated to the agents' coordination ability.

5.2 Gradualism

An interesting feature of the model is the nature of incompatible demands in equilibria featuring delay. Gradualism is a commonly observed feature of bargaining in which players gradually lower their demands, starting with very aggressive ones and ending with a compatible profile.[30] SPEs with delay accomodate gradualism in a natural way. This can be seen most easily in MPEs. The following corollary characterizes this feature.

Corollary 1 *If* (y, m) *is the outcome of a Markov perfect equilibrium with a delay of* $m - 1 > 0$ *periods then the incompatible demand profiles* z^t *for* $1 \leq t \leq m - 1$ *must satisfy*

$$z_i^t \geq \frac{(1 - \delta_{-i}^{m-t} y_{-i})(1 + k_i) - \delta_i^{m-t} y_i}{k_i}.$$

In words, the smallest (incompatible) demand that can arise in an MPE is higher the further away (in periods) it is made from the eventual agreement. Two separate features contribute to this. The obvious one is that for neither player to want to deviate to simply accepting the others implicit offer (by making a compatible demand) it must be that the offers are worse than accepting the delayed agreement. The longer the delay the worse the offers need to be, and therefore higher demands. The less obvious feature is that a bargainer may find it profitable to deviate to a lower but still incompatible demand profile that extracts a concession from the other player. To rule out such a deviation, the incompatible demands need to be even higher than the level required to rule out deviations to compatible profiles. Further, this threshold is higher the more periods that remain to agreement.

5.3 Simultaneous versus sequential demands

The importance of simultaneous versus sequential *demands* is, in a sense, a superficial one in the current setting. Instead, the two key feature are the following. First, the demand made by one agent is not just met by a decision to

[30] See, for instance, Backus, Blake, Larsen and Tadelis (2020).

accept or reject, but by a competing demand from the other agent. Second, once two incompatible demands arrive at the table, the decision to accept or reject (stick) is made simultaneously. The latter concession stage is where the agents's concerns about concession costs and patience combine to determine her bargaining strength, and simultaneity here is important. By contrast, requiring the demands to be made in some arbitrarily fixed order, has a lot less impact. For instance, all RP (or no-delay) SPE outcomes identified in propositions 5 and 3 continue to be supported by SPE, with sequential *demands* that follow some fixed order. The specification matches descriptive accounts of bilateral negotiations that associate a round of bargaining with two competing positions. See for instance, the evolution EU-UK positions on citizen's rights during Brexit negotiations in 2017 listed in Department for Exiting the European Union and Home Office (2017).

Negotiations that feature collective bargaining in North America also match the specification here, in that it involves multiple rounds with each round featuring a pair of competing offers and subsequent discussion. In a given round, the order of proposal is neither specified nor is given any importance. In particular, it is *not* the case that within a round a competing proposal is made only after the first is rejected. Further, the first round explicitly requires the simultaneous exchange of offers. These negotiations occur in a variety of settings, such as between a university administration and the relevant chapter of the American Association of University Professors, the governments of Canada and the United States over an updated Columbia River Treaty, and the Nuclear Regulatory Group and the Canadian Nuclear Safety Commission 2022.[31]

5.4 Other Related Literature

Ellingsen and Miettinen (2014) (henceforth EM) extend the static model of Ellingsen and Miettinen (2008) to a fairly involved dynamic model. Formalizations of Schelling's ideas are usually closely related to the Nash demand game. The EM model has elements of both the Nash demand game (simultaneous demands) and the generalized Rubinstein bargaining framework. As examples of

[31] See Rider University (2022), Government of British Columbia (2023) and The Professional Institute of the Public Service of Canada (2019).

the latter, (a) following demands that are more than compatible, a single responder is selected randomly to accept or reject the other's offer and (b) following a choice of flexibility by both bargainers, a single player is randomly selected to make an offer for that period. The key difference with the current formalization, however, is that in EM (as well as Ellingsen and Miettinen (2008)) commitment ability is exogenous and independent of the actual demands made by the players. It does not matter whether a bargainer is offered a lot of room to back down or none at all, her commitment ability is pinned down by an exogenous randomization device. This distinction is critical, since in the current study the strategic feature that resolves the bargaining problem, is precisely the ability of bargainers to affect each other's commitment ability by choosing appropriate demands.

The delay obtained in Markov perfect equilibrium in section 3.3 is neither the result of money burning as in Avery and Zemsky (1994) nor due to strategic uncertainty as in Friedenberg (2019). In a sense, as Sakovics (1993) puts it, the delay is wholly ritualistic and can be expected in settings where bargainers take their cues from norms or traditions that are perhaps optimal in some larger context but offer an inefficient prescription in the specific bargaining instance. Similar equilibria also arise in Perry and Reny (1993) and Sakovics (1993), who study a generalization of the Rubinstein model with less restriction on when offers can be made and responded to. A key finding in both is that allowing for simultaneous demands generates an acute multiplicity of equilibria including those with delay. While not their focus, the SPE with delay in these models feature a milder form of the gradualism that appears in the current study. The further away the anticipated agreement, the further apart the incompatible demands need to be to deter deviation to a compatible profile. As stated earlier, in the current study the incompatible demands need to be even further apart to rule out deviations to incompatible profiles. Compte and Jehiel (2004) provides a wholly different rationale for gradualism. Players always have access to outside options whose values depend on past offers. If more favourable offers increase the value of the opponent's outside option, then bargainers find it optimal to lower their demand gradually in equilibrium.

A Appendix

Lemma 4 *σ cannot be an SPE in the general model, if for some $h \in H$, $\sigma(h) = z$ such that $z_i = 1$ and $z_{-i} = 0$ for some $i \in \{1, 2\}$.*

Proof. Suppose under σ, in the subgame $g(h)$, the two players make the compatible demands $z_i = 1$ and $z_{-i} = 0$, and player $-i$ obtains a payoff of $u_{-i}(0) = 0$. The highest payoff player i could get if bargaining broke down this period is $\delta_i u_i(1)$. Notice that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1)$ is a continuous (decreasing) function of $\hat{z}_{-i}$. It takes a value of $u_i(1)$ at $\hat{z}_{-i} = 0$, which is strictly greater than $\delta_i u_i(1)$. Therefore there exists $\hat{z}_{-i} > 0$ such that $u_i(1 - \hat{z}_{-i}) - c_i(z_i + \hat{z}_{-i} - 1) > \delta_i u_i(1)$. Now, if player $-i$ were to deviate to this $\hat{z}_{-i}$ instead of demanding 0, then in the subsequent concession game the dominance solvable outcome would involve player i playing A and $-i$ playing S. Since this is a profitable deviation, the strategy profile σ cannot be an SPE.

■

Proof for Lemma 2.

Without loss of generality, set $i = 1$. Now note that bargaining failure in period t leads to $g(h^t)$ beginning in the next period. Since σ is an SPE with maximum delay n^*, and by lemma 1, the outcome (x, m) of this subgame must satisfy $x \in B^{n^*}$ and $m \leq n^* + 1$. Suppose one such continuation outcome is given by $(\tilde{z}, n)$. Now consider a deviation $\hat{z}_2$ from the compatible profile z which satisfies both inequalities 1 and 2 for this continuation profile.

Table 3: Augmented Concession Game following deviation $\hat{z}_2$ from Profile z

	A	S
A	$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$	$1 - \hat{z}_2 - k_1(z_1 + \hat{z}_2 - 1), \hat{z}_2$
S	$z_1, 1 - z_1 - k_2(z_1 + \hat{z}_2 - 1)$	$\delta_1^n \tilde{z}_1, \delta_2^n \tilde{z}_2$

The deviation leads to the augmented game above in the concession stage, with (S, S) yielding a discounted payoff consistent with the continuation outcome $(\tilde{z}, n)$. Due to inequality 1, in this concession stage S strictly dominates A for player 2. Inequality 2 in turn ensures that given player 2's choice of S, player 1 strictly prefers to play A. In other words, the unique dominance solvable outcome in the augmented concession game is (A, S). Furthermore this outcome gives player 2 a strictly higher payoff than z_2. So, if there exists a $\hat{z}_2$ such that no matter what the continuation profile (consistent with σ being an SPE with maximum delay n^*) the two inequalities above are always satisfied, then $\hat{z}_2$ is a profitable deviation from z and therefore σ is not an SPE. ■

Proof for Lemma 3.

The necessity of $x \in B^*$ follows by definition. By way of contradiction, suppose that (x, t) is an SPE outcome with $t > 1$, $x \in B^*$ and

$$\delta_1^{t-1} x_1 < \frac{1 - \delta_2 z_2^*}{1 + k_2}$$

where $z_2^* = \sup_{z \in B^*} z_2$. It then suffices to show that player 1 is better off deviating from her first period incompatible demand.

Consider the first period incompatible demand profile, z^1 in such an SPE. It must be that $z_i^1 \geq 1 - \delta^{t-1} x_{-i}$ for $i \in \{1, 2\}$. Otherwise player i could profitably deviate to making the compatible demand $1 - z_{-i}^1$ in period 1.

Fix some continuation payoff profile w. Then the set of incompatible demand profiles y for which player 2 is indifferent between A and S, conditional on player 1 choosing S, is given by the the equation $1 - y_1 - k_2(y_1 + y_2 - 1) = w_2$. Rewrite this as $y_1 = 1 - \frac{k_2}{1+k_2} y_2 - \frac{w_2}{1+k_2}$. The best continuation payoff for player 2 is $\delta_2 z_2^*$. Let

$$y_1^*(y_2) = 1 - \frac{k_2}{1 + k_2} y_2 - \frac{\delta_2 z_2^*}{1 + k_2}.$$

Notice that for any incompatible demand profile y, with $y_1 < y_1^*(y_2)$, player 2 strictly prefers A to S, conditional on 1 choosing S, if her continuation payoff is $\delta_2 z_2^*$. Further, since any SPE continuation payoff w_2 is no greater than $\delta_2 z_2^*$, following incompatible profile y with $y_1 < y_1^*(y_2)$, player 2 strictly prefers A to S, conditional on 1 choosing S, *for any SPE continuation profile.*

Given continuation payoff profile w, the set of incompatible demand profiles y for which player 1 is indifferent between A and S, conditional on player 2 choosing S, satisfies the equation $1 - y_2 - k_1(y_1 + y_2 - 1) = w_1$. Rewrite this as

$y_1 = (1-y_2)\frac{1+k_1}{k_1} - w_1\frac{1+k_1}{k_1}$. Let

$$y_1^{**}(y_2) = (1-y_2)\frac{1+k_1}{k_1}.$$

Then for any incompatible demand profile y with $y_1 > y_1^{**}(y_2)$, player 1 strictly prefers to S to A, *for any SPE continuation profile.*

Return to the premise of player 1's SPE payoff $\delta_1^{t-1}x_1$ and first period incompatible profile z^1. By the inequalities derived above, if for all $z_2^1 \geq 1-\delta^{t-1}x_1$, the inequalities $y_1^*(z_2^1) > \delta_1^{t-1}x_1$ and $y_1^*(z_2^1) > y_1^{**}(z_2^1)$ hold, then a contradiction obtains. Player 1 could then profitably deviate in period 1 to making an incompatible demand $y_1^*(z_2^1) > \hat{z}_1^1 > \max\{y_1^{**}(z_2^1), \delta_1^{t-1}x_1\}$ and force player 2 to concede, no matter the SPE continuation profile.

Next observe that $y_1^*(1) = \frac{1-\delta_2 z_2^*}{1+k_2}$. Since $\delta_1^{t-1}x_1 < \frac{1-\delta_2 z_2^*}{1+k_2}$ (by assumption) and y_1^* is a decreasing function, it follows that $y_1^*(z_2^1) > \delta_1^{t-1}x_1$ for all $z_2^1 \geq 1-\delta^{t-1}x_1$. Since y_1^{**} is also a decreasing linear function and $y_1^{**}(1) = 0$, to obtain the contradiction, it suffices to show that $y_1^*(1-\delta_1^{t-1}x_1) > y_1^{**}(1-\delta_1^{t-1}x_1)$. Some computation shows that $y_1^*(1-v) \leq y_1^{**}(1-v)$ requires

$$v \geq \frac{k_1(1-\delta_2 z_2^*)}{1+k_1+k_2}.$$

Since $\delta_1^{t-1}x_1 < \frac{1-\delta_2 z_2^*}{1+k_2}$, the contradiction would obtain if

$$\frac{1-\delta_2 z_2^*}{1+k_2} < \frac{k_1(1-\delta_2 z_2^*)}{1+k_1+k_2}.$$

The inequality indeed follows from the assumption of $k_2(k_1-1) > 1$. This concludes the proof for $i=1$. A symmetric argument works for $i=2$.

■

Proof for Proposition 2.

Let $O^{SPEd} = \{(z,t) \in O^{SPE} | t > 1\}$ collect all SPE outcomes that feature delay and $D = \{(w_1, w_2) | w_i = \delta^t z_i \textit{ for } i \in \{1,2\} \textit{ and } (z,t) \in O^{SPEd}\}$ be the set of continuation payoffs such SPE with delay generate. Let $w_i^m = \inf_{w \in D} w_i$. Recall that $B^* = \{z | (z,t) \in O^{SPE}\}$. Let $z_i^* = \sup_{z \in B^*} z_i$.

I first show that there cannot be a deviation $\hat{z}_2 > 1 - z_1^*$ such that

$$1 - z_1^* - k_2(z_1^* + \hat{z}_2 - 1) < \min\{w_2^m, \delta_2(1 - z_1^*)\} \tag{14}$$

and

$$1 - \hat{z}_2 - k_1(z_1^* + \hat{z}_2 - 1) > \delta_1 z_1^*. \tag{15}$$

The right hand side (RHS) of inequality 14 gives the worst SPE continuation payoff for player 2, while the RHS of inequality 15 gives the best SPE continuation payoff for player 1. The existence of such a $\hat{z}_2$ means that player 2 has a profitable deviation from the efficient profile $(z_1^*, 1 - z_1^*)$. This is because following such a deviation, in the resulting concession game player 2's choice of S strictly dominates A, *irrespective of the SPE continuation payoff*, due to inequality 14. Further, due to inequality 15, player 1 strictly prefers A over S, in the face of 2 choosing S. In other words, following the deviation to $\hat{z}_2$, the dominance solvable outcome of the concession game is (A, S) and brings 2 the higher payoff of $\hat{z}_2$, no matter the continuation SPE profile. Such a $\hat{z}_2$ rules out $(z_1^*, 1 - z_1^*) \in B^*$ and by continuity rules out $z_1^* = \sup_{z \in B^*} z_1$.

Inequality 14 simplifies to

$$\hat{z}_2 > \frac{(1 - z_1^*)(1 + k_2) - \min\{w_2^m, \delta_2(1 - z_1^*)\}}{k_2}$$

while 15 simplifies to

$$\hat{z}_2 < 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

Therefore $z_1^* = \sup_{z \in B^*} z_1$ requires

$$\frac{(1 - z_1^*)(1 + k_2) - \min\{w_2^m, \delta_2(1 - z_1^*)\}}{k_2} \geq 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

By proposition 3, $w_2^m \geq \frac{1 - \delta_1 z_1^*}{1 + k_1}$. Then the relevant inequality is

$$\frac{(1 - z_1^*)(1 + k_2) - \min\{\frac{1 - \delta_1 z_1^*}{1 + k_1}, \delta_2(1 - z_1^*)\}}{k_2} \geq 1 - \frac{(k_1 + \delta_1)z_1^*}{1 + k_1}.$$

There are two cases to consider. If $\frac{1-\delta_1 z_1^*}{1+k_1} \geq \delta_2(1-z_1^*)$ then the inequality reduces to

$$\frac{z_1^*}{1-z_1^*} \leq \frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2}. \tag{16}$$

Alternatively, if $\frac{1-\delta_1 z_1^*}{1+k_1} < \delta_2(1-z_1^*)$ then the inequality reduces to

$$\frac{z_1^*}{1-z_1^*} < \frac{k_1}{(1+k_2)(1-\delta_1)}.$$

Now, $\frac{1-\delta_1 z_1^*}{1+k_1} \geq \delta_2(1-z_1^*)$ itself simplifies to

$$\frac{z_1^*}{1-z_1^*} \geq \frac{\delta_2(1+k_1)-1}{1-\delta_1}.$$

Since in this case, inequality 16 emerges, it must be that

$$\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2} \geq \frac{\delta_2(1+k_1)-1}{1-\delta_1}.$$

It turns out that this inequality is equivalent to

$$\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2} \geq \frac{k_1}{(1+k_2)(1-\delta_1)}.$$

This generates the required expression

$$\frac{z_1^*}{1-z_1^*} \leq \max\left\{\frac{1-\delta_2}{1-\delta_1}\frac{1+k_1}{k_2}, \frac{k_1}{(1+k_2)(1-\delta_1)}\right\}.$$

■

Lemma 5 *The strategy profile σ described in Construction 1 is an SPE if*

$$\frac{1-\delta_1}{1-\delta_2}\frac{k_2}{1+k_1} \leq \frac{z_2}{z_1} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1+k_2}{k_1}.$$

Proof. The payoff to player i from σ at any subgame $g(h)$ with $h \in H$ is simply z_i. A lower demand would only lower the payoff. A higher demand would lead to either (S,S) and a continuation payoff of $\delta_i z_i$ or (A_i, S_{-i}) leading to a payoff strictly lower than z_i due to the resulting concession cost. Therefore no player has an incentive to deviate in the demand stage of any period.

To verify subgame perfection, therefore, it is sufficient to show that in the concession stage game following an incompatible demand profile $(\hat{z}_i, z_{-i})$, if (S, S) is not a Nash equilibrium then (A_i, S_{-i}) is. To establish this result, in turn, it is sufficient to show the following,

$$1 - \hat{z}_i - k_{-i}(\hat{z}_i + z_{-i} - 1) > \delta_{-i} z_{-i} \Rightarrow 1 - z_{-i} - k_i(\hat{z}_i + z_{-i} - 1) > \delta_i z_i$$

which is equivalent to

$$1 - \frac{\delta_{-i} z_{-i} + k_{-i} z_{-i}}{1 + k_{-i}} > \hat{z}_i \Rightarrow \frac{(1 - z_{-i})(1 + k_i - \delta_i)}{k_i} > \hat{z}_i.$$

A sufficient condition for this is simply

$$\begin{aligned} \frac{(1 - z_{-i})(1 + k_i - \delta_i)}{k_i} &> 1 - \frac{\delta_{-i} z_{-i} + k_{-i} z_{-i}}{1 + k_{-i}} \\ \Leftrightarrow \frac{(1 - z_{-i})(1 - \delta_i)}{k_i} &> \frac{z_{-i}(1 - \delta_{-i})}{1 + k_{-i}} \\ \Leftrightarrow \frac{1 - \delta_i}{1 - \delta_{-i}} \frac{1 + k_{-i}}{k_i} &> \frac{z_{-i}}{1 - z_{-i}}. \end{aligned}$$

Requiring the above inequalities to hold for $i \in \{1, 2\}$ make them equivalent to

$$\frac{1 - \delta_1}{1 - \delta_2} \frac{k_2}{1 + k_1} \leq \frac{z_2}{z_1} \leq \frac{1 - \delta_1}{1 - \delta_2} \frac{1 + k_2}{k_1}.$$

■

Proof for Proposition 3. Lemma 4 above establishes that, even in the general model, compatible demand profiles in which one player demands the entire surplus cannot arise in an SPE. This combined with lemma 1 implies that if (z, t) is the outcome of an SPE then $d(z) = 0$ and $z_i \in (0, 1)$ for $i \in \{1, 2\}$. This in turn means that if σ is a renegotiation-proof SPE with outcome (z, t) then $t = 1$. To see why, suppose instead that $t > 1$. Then

$$\psi(\sigma; h^0) = (\delta_1^{t-1} z_1, \delta_2^{t-1} z_2) \ll (z_1, z_2) = \psi(\sigma, \tilde{h}^t),$$

where $\tilde{h}^t$ is the history that occurs on the equilibrium path with the $t-1$ periods of incompatible demands with neither player conceding in the subsequent concession games. Therefore by proposition 1, inequality 5 is a necessary condition for renegotiation-proof SPE outcomes. Lemma 5 establishes sufficiency by constructing stationary SPE strategies with outcome (z, t) for any z satisfying inequality 5 and $t = 1$. Fix one such z and its corresponding stationary SPE strategy profile, σ. Notice that σ satisfies renegotiation-proofness since by construction $\psi(\sigma; h) = (z_1, z_2)$ for all $h \in H$. ■

Lemma 6 *If $\delta_j \geq \delta_{-j}$ then*

$$\frac{1-\delta_j^n}{1-\delta_{-j}^n} \leq \frac{1-\delta_j^{n+1}}{1-\delta_{-j}^{n+1}}.$$

Proof.

$$\begin{aligned}
\frac{1-\delta_j^n}{1-\delta_{-j}^n} \leq \frac{1-\delta_j^{n+1}}{1-\delta_{-j}^{n+1}} \Leftrightarrow & \frac{1-\delta_{-j}^{n+1}}{1-\delta_{-j}^n} \leq \frac{1-\delta_j^{n+1}}{1-\delta_j^n} \\
\Leftrightarrow & \frac{1+\delta_{-j}+\delta_{-j}^2+\ldots+\delta_{-j}^n}{1+\delta_{-j}+\delta_{-j}^2+\ldots+\delta_{-j}^{n-1}} \leq \frac{1+\delta_j+\delta_j^2+\ldots+\delta_j^n}{1+\delta_j+\delta_j^2+\ldots+\delta_j^{n-1}} \\
\Leftrightarrow & \frac{\delta_{-j}^n}{1+\delta_{-j}+\delta_{-j}^2+\ldots+\delta_{-j}^{n-1}} \leq \frac{\delta_j^n}{1+\delta_j+\delta_j^2+\ldots+\delta_j^{n-1}} \\
\Leftrightarrow & \frac{1}{\delta_j^n}+\frac{1}{\delta_j^{n-1}}+\ldots+\frac{1}{\delta_j} \leq \frac{1}{\delta_{-j}^n}+\frac{1}{\delta_{-j}^{n-1}}+\ldots+\frac{1}{\delta_{-j}} \\
\Leftrightarrow & \delta_j \geq \delta_{-j}.
\end{aligned}$$

■

Markov Perfect Equilibria

Proof for Proposition 4. Consider a Markov strategy profile with outcome $(y, t+1)$ where $t \geq 1$ (features delay). For this to arise in equilibrium, among other requirements, neither player must have a profitable deviation from the first period incompatible demand profile, say z. The condition $\delta_i^t y_i > 1 - z_{-i}$ for $i \in \{1, 2\}$ rules out profitable deviations to compatible demand profiles. Let the remaining set of incompatible demand profiles be $D(y, t+1)$. Notice that

$(1,1) \in D(y,t+1)$. For $z \in D(y,t+1)$ to be the first period incompatible profile in an equilibrium with outcome $(y,t+1)$, it must further be that neither player has a profitable deviation to yet another incompatible profile. The next argument shows that if such a deviation exists from the profile $(1,1)$ then it also does for any $z \in D(y,t+1)$.

Suppose $z=(1,1)$. Then a profitable deviation by player i to an incompatible profile $(\tilde{z}_i,1)$ requires $\tilde{z}_i > \delta_i^t y_i$ and (S_i, A_{-i}) to be the dominance solvable outcome in the subsequent concession game. For the latter to be true it must be that $1-\tilde{z}_i - k_{-i}(\tilde{z}_i+1-1) > \delta_{-i}^t y_{-i}$ and $1-1-k_i(\tilde{z}_i+1-1) < \delta_i^t y_i$. So long as

$$\frac{1-\delta_{-i}^t y_{-i}}{1+k_{-i}} > \delta_i^t y_i$$

such a $\tilde{z}_i$ can be found. Suppose this is true. Then consider any $z \in D(y,t+1)$. To prove that a similar deviation exists from z it suffices to show that for $z_i^*(z_{-i})$ and $z_i^{**}(z_{-i})$ that solve $1-z_i^*-k_{-i}(z_i^*+z_{-i}-1)=\delta_{-i}^t y_{-i}$ and $1-z_{-i}-k_i(z_i^{**}+z_{-i}-1)=\delta_i^t y_i$, respectively, $z_i^* > z_i^{**}$. Since then any $\tilde{z}_i \in (z_i^{**}, z_i^*)$ would be a profitable deviation. Note that both z_i^* and z_i^{**} are linear and strictly decreasing in z_{-i}. It has already been shown that $z_i^*(1) > z_i^{**}(1)$. Therefore it suffices to show that if $z_i^*(z'_{-i}) = z_i^{**}(z'_{-i})$ then $z'_{-i} < 1-\delta_i^{t-1} y_i$. Indeed, $z_i^*(z'_{-i}) = z_i^{**}(z'_{-i})$ implies that

$$z'_{-i} = \frac{1+k_{-i}}{1+k_1+k_2}(1-\delta_i^t y_i) + \frac{k_i}{1+k_1+k_2}\delta_{-i}^t y_{-i} < 1-\delta_i^t y_i.$$

Therefore for all $z \in D(y,t+1)$, $z_i^*(z_{-i}) > z_i^{**}(z_{-i})$, which in turn implies that for any such z player i has a profitable deviation $\tilde{z}_i \in (z_i^{**}(z_{-i}), z_i^*(z_{-i}))$.

Necessity

Consider a Markov strategy profile with outcome $(y,t+1)$ where $t \geq 1$. It has been shown above that for this to be an equilibrium neither player should have a profitable deviation from a first period incompatible demand of $(1,1)$. The largest demand player i can make that would lead to the dominance solvable outcome of (S_i, A_{-i}) in the concession game is given by $\frac{1-\delta_{-i}^t y_{-i}}{1+k_{-i}}$. To rule out the profitable deviation from $(1,1)$, this must be lower than $\delta_i^t y_i$. This yields two inequalities

$$\frac{1-\delta_2^t y_2}{1+k_2} \leq \delta_1^t y_1 \qquad and \qquad \frac{1-\delta_1^t y_1}{1+k_1} \leq \delta_2^t y_2.$$

Since $y_1 + y_2 = 1$ by lemma 1, the inequalities can be rewritten in terms of y_1 alone,

$$\frac{1-\delta_2^t}{\delta_1^t(1+k_2)-\delta_2^t} \leq y_1 \qquad and \qquad y_1 \leq \frac{\delta_2^t(1+k_1)-1}{\delta_2^t(1+k_1)-\delta_1^t}. \tag{17}$$

For such a y_1 to exist it must be that

$$\frac{1-\delta_2^t}{\delta_1^t(1+k_2)-\delta_2^t} \leq \frac{\delta_2^t(1+k_1)-1}{\delta_2^t(1+k_1)-\delta_1^t}$$

which simplifies to the condition

$$\frac{k_1}{\delta_1^t} + \frac{k_2}{\delta_2^t} \leq k_1 + k_2 + k_1 k_2. \tag{18}$$

Further, the inequalities in 17 imply

$$\frac{1-\delta_1^t}{\delta_2^t(1+k_1)-1} \leq \frac{y_2}{y_1} \leq \frac{\delta_1^t(1+k_2)-1}{1-\delta_2^t}. \tag{19}$$

Inequalities 18 and 19 followed simply from requiring no player to have a profitable deviation from making incompatible demands in the t initial periods of impasse for the outcome $(y, t+1)$. The final necessary condition follows from requiring $(y, 1)$ to be a no-delay MPE outcome.

Suppose in an MPE, conditional on an impasse in period 1, the outcome is $(\tilde{z}, t+1)$ (with $t \geq 0$) in the subgame starting in period 2. What is the best efficient equilibrium outcome that can arise in period 1 for player 1 given this continuation play? The answer lies in solving for z_1 in the following pair of equations

$$1 - z_1 - k_2(z_1 + z_2 - 1) = \delta_2^{t+1}(1-\tilde{z}_1),$$
$$1 - z_2 - k_1(z_1 + z_2 - 1) = \delta_1^{t+1}\tilde{z}_1.$$

Solving for z_1 gives

$$z_1 = \frac{(1+k_1)(1-\delta_2^{t+1}(1-\tilde{z}_1)) + k_2\delta_1^{t+1}\tilde{z}_1}{1+k_1+k_2}.$$

A compatible demand profile with a higher share to player 1 than above would allow player 2 to profitably deviate and extract a concession from 1. Notice that z_1 as computed above is increasing in $\tilde{z}_1$ and $\frac{\partial z_1}{\partial \tilde{z}_1} < 1$. Next let C^M be the set of compatible demand profiles that can arise in some Markov perfect equilibrium and $z_1^* = \sup_{z \in C^M} z_1$. Then z_1^* must be the best efficient equilibrium outcome in period 1 for player 1 for some equilibrium continuation play outcome $(\tilde{z}, t+1)$. Clearly $\tilde{z}_1 \leq z_1^*$. Further since $\frac{\partial z_1}{\partial \tilde{z}_1} < 1$, it must be that $z_1^* \leq z_1^{*t+1}$ where

$$z_1^{*t+1} = \frac{(1+k_1)(1-\delta_2^{t+1}(1-z_1^{*t+1})) + k_2\delta_1^{t+1}z_1^{*t+1}}{1+k_1+k_2}$$

for some t.

Solving for z_1^{*t+1} gives

$$z_1^{*t+1} = \frac{(1+k_1)(1-\delta_2^{t+1})}{(1+k_1)(1-\delta_2^{t+1}) + k_2(1-\delta_1^{t+1})}.$$

An analogous computation for z_2^* gives

$$z_2^{*t+1} = \frac{(1+k_2)(1-\delta_1^{t+1})}{(1+k_2)(1-\delta_1^{t+1}) + k_1(1-\delta_2^{t+1})}.$$

It then follows that

$$\frac{z_i^*}{1-z_i^*} \leq \frac{z_i^{*t+1}}{1-z_i^{*t+1}} = \frac{(1+k_i)(1-\delta_{-i}^{t+1})}{k_{-i}(1-\delta_i^{t+1})} \tag{20}$$

for some t.

Lemma 6 shows

$$\delta_j \geq \delta_{-j} \Leftrightarrow \frac{1-\delta_j^n}{1-\delta_{-j}^n} \leq \frac{1-\delta_j^{n+1}}{1-\delta_{-j}^{n+1}}.$$

This implies that for a given $i \in \{1,2\}$, the relevant t in equation 20 is either 1 or the maximum delay permitted in an MPE. Therefore if $(z, t+1)$ is an MPE outcome it must be that

$$\frac{k_2}{1+k_1}\min\left\{\frac{1-\delta_1}{1-\delta_2}, \frac{1-\delta_1^{n^{MB}+1}}{1-\delta_2^{n^{MB}+1}}\right\} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1}\max\left\{\frac{1-\delta_1}{1-\delta_2}, \frac{1-\delta_1^{n^{MB}+1}}{1-\delta_2^{n^{MB}+1}}\right\}$$

where n^{MB} is the largest n that satisfies inequality 18.

Sufficiency Select some $(z, t+1) \in O^{AMP}$ such that

$$\frac{k_2}{1+k_1}\frac{1-\delta_1^l}{1-\delta_2^l} \leq \frac{z_2}{z_1} \leq \frac{1+k_2}{k_1}\frac{1-\delta_1^l}{1-\delta_2^l} \tag{21}$$

for some $1 \leq l \leq t+1$.

Consider the following strategy profile. For the first t periods both bargainers make a demand of 1. Further, for $n \in \{0, 1, 2, \dots\}$, both bargainers make a demand of 1 in all periods $t+1+nl+m$ where $m \in \{1, 2, \dots, l-1\}$. In periods $t+1+nl$, player i demands z_i for $i \in \{1, 2\}$. In any concession stage game, if it is dominance solvable, the bargainers play the resulting unique equilibrium. In any concession stage game that follows a period $t+1+nl+m$ incompatible demand profile wherein player i makes a demand greater than z_i while $-i$'s demand remains at z_{-i}, and is not dominance solvable, the profile (A_i, S_{-i}) is played.

To see that this Markov strategy profile satisfies subgame perfection, consider period 1, which features the incompatible demand profile $(1, 1)$ and player i expects a present discounted continuation payoff of $\delta_i^t z_i$. Clearly neither player is better off unilaterally deviating to a compatible demand profile. A unilateral deviation by player i to a still incompatible profile yields one of two outcomes. Either the dominance solvable outcome in the resulting concession game is (S, S) which is not profitable, or it is (S_i, A_{-i}). For the latter to occur it must be that $\tilde{z}_i$, the demand i deviates to, is no greater than $\frac{1-\delta_{-i}^t z_{-i}}{1+k_{-i}}$. For this to be profitable requires further that $\tilde{z}_i > \delta_i^t z_i$. But this is not possible since $(z, t+1) \in O^{AMP}$, which implies that $\frac{1-\delta_{-i}^t z_{-i}}{1+k_{-i}} \leq \delta_i^t z_i$. Note further that $\frac{1-\delta_{-i}^t z_{-i}}{1+k_{-i}} \leq \delta_i^t z_i$ implies $\frac{1-\delta_{-i}^{t'} z_{-i}}{1+k_{-i}} \leq \delta_i^{t'} z_i$ for all $1 \leq t' \leq t$ and therefore for all $1 \leq t' \leq l-1$ too. As a result, by an analogous argument, neither player has an incentive to deviate in any of the first t periods as well as all periods $t+1+nl+m$ with $m \in \{1, 2, \ldots, l-1\}$, that feature the incompatible demand profile $(1, 1)$.

It remains to show that neither player has a profitable deviation from the compatible demand profile z in the periods $t+1+nl$. Bargainer i faces a present discounted continuation payoff of $\delta^l z_i$. The only way player i can profitably deviate is by ensuring that in the concession game the dominance solvable outcome is (S_i, A_{-i}). This requires,

$$1 - \tilde{z}_i - k_{-i}(\tilde{z}_i + z_{-i} - 1) > \delta_{-i}^l z_{-i} \qquad \textit{and} \qquad 1 - z_{-i} - k_i(\tilde{z}_i + z_{-i} - 1) < \delta_i^l z_i$$

which simplify to

$$\hat{z}_i < 1 - \frac{(k_{-i} + \delta_{-i}^l) z_{-i}}{1 + k_{-i}}, \qquad \textit{and} \qquad \hat{z}_i > \frac{(1 - z_{-i})(1 + k_i - \delta_i^l)}{k_i}.$$

Such a $\tilde{z}_i$ can exist only if

$$\begin{aligned} & \frac{(1 - z_{-i})(1 + k_i - \delta_i^l)}{k_i} < 1 - \frac{(k_{-i} + \delta_{-i}^l) z_{-i}}{1 + k_{-i}} \\ \Rightarrow & \frac{(1 - z_{-i})(1 - \delta_i^l)}{k_i} < \frac{z_{-i}(1 - \delta_{-i}^l)}{1 + k_{-i}} \\ \Rightarrow & \frac{1 - z_{-i}}{z_{-i}} < \frac{1 - \delta_{-i}^l}{1 - \delta_i^l} \frac{k_i}{1 + k_{-i}}. \end{aligned}$$

This is not possible since by inequality 21 ■

Proof for Proposition 5. A stationary MPE must feature either immediate agreement or perpetual delay. Perpetual delay is ruled out since either player would deviate in the first period to making an arbitrarily small demand. This would either lead to a compatible demand profile, or if incompatible force the opponent to concede. Therefore stationary MPEs feature no delay. The result then follows from lemma 5 and proposition 1. ■

Lemma 7 *For $i \in \{1, 2\}$ and $n^* \in \mathbb{N}$, following equations 8, 9 and 10,*
(a) $\tilde{z}_{-i}^{n^*}$ is a well defined function with $-1 < \frac{\partial \tilde{z}_{-i}^{n^*}}{\partial z_i} < 0$,

(b) $\tilde{\tilde{z}}^{n^*}_{-i}$ *is a well defined function with* $\frac{\partial \tilde{\tilde{z}}^{n^*}_{-i}}{\partial z_i} < -1$,
(c) $z_i^{Mn^*}$ *is well defined.*

Proof. (a) Since c_{-i} is unbounded above, $\tilde{z}^{n^*}_{-i}$ is indeed well defined for all $z_i \in [0,1]$. Further by the implicit function theorem it is a decreasing function with slope

$$\frac{\partial \tilde{z}^{n^*}_{-i}}{\partial z_i} = -\frac{(1-\delta_{-i}^{n^*+1})u'_{-i}(1-z_i)}{c'_{-i}(z_i + \tilde{z}^{n^*}_{-i} - 1)} - 1 < -1.$$

(b) Again by the implicit function theorem, $\tilde{\tilde{z}}^{n^*}_{-i}$ is well defined, decreasing and with slope

$$\frac{\partial \tilde{\tilde{z}}^{n^*}_{-i}(z_i)}{\partial z_i} = -\frac{\delta_i u'_i(z_i) + c'_i(z_i + \tilde{\tilde{z}}^{n^*}_{-i} - 1)}{u'_i(1-\tilde{\tilde{z}}^{n^*}_{-i}) + c'_i(z_i + \tilde{\tilde{z}}^{n^*}_{-i} - 1)} > -1$$

by the concavity of u_i.
(c) Note that $\tilde{z}^{n^*}_{-i}(1) = 0$ while $\tilde{\tilde{z}}^{n^*}_{-i}(1) > 0$. Also, $\tilde{z}^{n^*}_{-i}(0) > 1$ while $\tilde{\tilde{z}}^{n^*}_{-i}(0) = 1$. Therefore the function $\tilde{z}^{n^*}_{-i}(z_i) - \tilde{\tilde{z}}^{n^*}_{-i}(z_i)$ is positive at $z_i = 0$, negative at $z_i = 1$, continuous and (from the slope inequalities above) strictly decreasing over the interval $[0,1]$. By the intermediate value theorem it follows that $z_i^{Mn^*}$ is well defined and unique. ■

Proof for Proposition 6. Let $z_i^{n^*} = \sup_{z \in B^{n^*}} z_i$. Then there cannot exist a deviation $\hat{z}_2 > 1 - z_1^{n^*}$ such that

$$u_2(1 - z_1^{n^*}) - c_2(z_1^{n^*} + \hat{z}_2 - 1) < \delta_2^{n^*+1} u_2(1 - z_1^{n^*}) \tag{22}$$

and

$$u_1(1 - \hat{z}_2) - c_1(z_1^{n^*} + \hat{z}_2 - 1) > \delta_1 u_1(z_1^{n^*}). \tag{23}$$

To see why, suppose that $\sigma(h^0) = (z_1^{n^*}, 1 - z_1^{n^*})$ and there exists $\hat{z}_2$ that satisfies the inequalities above. Then it must be that

$$u_2(1 - z_1^{n^*}) - c_2(z_1^{n^*} + \hat{z}_2 - 1) < \delta_2^{t+1} u_2(z_2)$$

and

$$u_1(1 - \hat{z}_2) - c_1(z_1^{n^*} + \hat{z}_2 - 1) > \delta_1^{t+1} u_1(z_1)$$

for any outcome $(u(z), t)$ of an SPE with maximum delay n^*, since for all such

$(u(z), t)$, it follows that $z_1 \leq z_1^{n^*}$ and $z_2 = 1 - z_1 > 1 - z_1^{n^*}$ and $1 \leq t + 1 \leq n^* + 1$. In other words, irrespective of the continuation strategy profile, following such a deviation, in the resulting concession stage game, the dominance solvable outcome would be (A, S), giving player 2 the payoff $u_2(\hat{z}_2)$ which is strictly greater than $u_2(1 - z_1^{n^*})$. Therefore, if such a deviation were to exist then $(z_1^{n^*}, 1 - z_1^{n^*}) \notin B^{n^*}$. The same argument ensures that $z \notin B^{n^*}$ for z arbitrarily close to $(z_1^{n^*}, 1 - z_1^{n^*})$, which in turn contradicts $z_1^{n^*} = \sup_{z \in B^{n^*}} z_1$.

Lemma 7 shows that $\tilde{z}_2^{n^*}$ and $\tilde{\tilde{z}}_2^{n^*}$ are well defined functions with $\tilde{z}_2^{n^*}(z_1) - \tilde{\tilde{z}}_2^{n^*}(z_1)$ strictly decreasing over the interval $[0, 1]$ and equal to zero at $z_1^{Mn^*}$. Now, it cannot be that $\tilde{\tilde{z}}_2^{n^*}(z_1^{n^*}) > \tilde{z}_2^{n^*}(z_1^{n^*})$ since then a deviation that satisfies inequalities 22 and 23 would exist; any $\hat{z}_2 \in (\tilde{z}_2^{n^*}(z_1^{n^*}), \tilde{\tilde{z}}_2^{n^*}(z_1^{n^*}))$ would suffice. Since $\tilde{\tilde{z}}_2^{n^*}(z_1) > \tilde{z}_2^{n^*}(z_1)$ for any $z_1 > z_1^{Mn^*}$, it must be that $z_1^{n^*} \leq z_1^{Mn^*}$. A symmetric argument establishes that $z_2^{n^*} \leq z_2^{Mn^*}$. ∎

Proof for Proposition 7.

Necessity

By lemma 1, any SPE at any history $h \in H$ will involve exactly compatible demands or incompatible ones followed by (S, S). SPE that further satisfy renegotiation-proofness cannot permit delay. To see this, consider a strategy profile, σ with outcome (y, t) where $t > 1$. By lemma 1, $y_i = u_i(z_i)$ with $d(z) = 0$. By lemma 4, $y_i > 0$. Now, $c(\sigma; h^0) = (\delta_1^t y_1, \delta_2^t y_2)$ while $c(\sigma; h^t) = (y_1, y_2)$. Since $c(\sigma; h^t) \gg c(\sigma; h^0)$, σ is not renegotiation-proof. This concludes the argument for why $t = 1$ if (y, t) is the outcome of a renegotiation-proof SPE in the general model. The rest follows from Proposition 6.

Sufficiency

Fix some z such that $d(z) = 0$ and $z_i \leq z_i^M$ for $i \in \{1, 2\}$. Consider the following stationary strategy profile, σ. For all $h^t \in H$, $\sigma_i(h^t) = z_i$. If player i in period t deviates to making a higher demand, $\hat{z}_i > z_i$, then in the concession stage game (S, S) is played if it is a Nash equilibrium and otherwise (A_i, S_{-i}) is played. For all other $h \in H'$ some pure strategy Nash equilibrium of the concession stage game is played.

Given the strategy profile σ, it is clear that making a lower demand at any period is never profitable. Making a higher demand for player i also yields her a lower payoff, since it either leads to (S, S) in the concession game and a continuation payoff of $\delta_i u_i(z_i)$ or (A_i, S_{-i}) with a payoff strictly less than $u_i(z_i)$ due to the concession cost. Hence no profitable deviation exists in any demand stage. To verify subgame perfection, therefore, it is sufficient to verify that following an incompatible demand profile $(z_i, \hat{z}_{-i})$, if (S, S) is not a Nash equilibrium then (S_i, A_{-i}) is. For this it is sufficient to show that $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$.

Recall that $\tilde{z}^0_{-i}(z_i)$ as defined in equation 8, with $n^* = 0$, corresponds to the smallest demand by $-i$ that leads to incompatibility and ensures that $-i$ prefers S over A in the subsequent concession stage game, assuming that in the next period the compatible profile z is announced. $\tilde{\tilde{z}}^0_{-i}(z_i)$, as defined in equation 9, with $n^* = 0$, in turn is the largest demand by $-i$ that leads to incompatibility and ensures that in the subsequent concession game, i prefers (A_i, S_{-i}) to (S, S), assuming that in the next period z is announced. So if $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ then following any incompatible demand $\hat{z}_{-i}$, if (A_i, S_{-i}) is a Nash equilibrium, then it must be that $\hat{z}_{-i} \leq \tilde{\tilde{z}}^0_{-i}(z_i) \leq \tilde{z}^0_{-i}(z_i)$ and therefore (S_i, A_{-i}) is a Nash equilibrium too. Since (A, A) is never a Nash equilibrium, this shows that with $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ if (S, S) is not a Nash equilibrium then (S_i, A_{-i}) must be.

Finally observe that $\tilde{z}^0_{-i}(z_i) \geq \tilde{\tilde{z}}^0_{-i}(z_i)$ since $z_i \leq z_i^M$. ■

Proof for Proposition 8. It follows from proposition 7 that

$$\xi(g^{c^n}) = \left\{ y = u(z) \middle| \frac{1 - z_1^{Mn}}{z_1^{Mn}} \leq \frac{z_2}{z_1} \leq \frac{z_2^{Mn}}{1 - z_2^{Mn}} \text{ and } d(z) = 0 \right\},$$

where the incompatible demand profile $(z_i^{Mn}, \hat{z}_{-i}^{Mn})$ for $i \in \{1, 2\}$ is characterized by the equations,

$$u_{-i}(1 - z_i^{Mn}) - c^n_{-i}(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1) = \delta_{-i} u_{-i}(1 - z_i^{Mn}) \tag{24}$$

$$u_i(1 - \hat{z}^n_{-i}(z_i^{Mn})) - c^n_i(z_i^{Mn} + \hat{z}^n_{-i}(z_i^{Mn}) - 1) = \delta_i u_i(z_i^{Mn}). \tag{25}$$

Set $z_i^{M*} = \lim_{n \to \infty} z_i^{Mn}$. Notice that since u_{-i} is bounded above and $c^{n'}_{-i}(0+) \to \infty$ as $n \to \infty$, it follows from equation 24 that $\lim_{n \to \infty} \hat{z}_{-i}(z_i^{Mn}) = 1 - \lim_{n \to \infty} z_i^{Mn} =$

$1 - z_i^{M*}$.

Now equations 24 and 25 together imply

$$\frac{(1-\delta_{-i})u_{-i}(1-z_i^{Mn})}{u_i(1-\hat{z}_{-i}^n(z_i^{Mn})) - \delta_i u_i(z_i^{Mn})} = \frac{c_{-i}^n(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c_i^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1)}.$$

Taking limits on both sides of this equation as $n \to \infty$ gives

$$\frac{(1-\delta_{-i})u_{-i}(1-z_i^{M*})}{(1-\delta_i)u_i(z_i^{M*})} = \lim_{n\to\infty} \frac{c_{-i}^n(z_i^{Mn} + \hat{z}_{-i}(z_i^{Mn}) - 1)}{c_i^n(z_i^{Mn} + \hat{z}_{-i}^n(z_i^{Mn}) - 1)}.$$

The right hand side is equal to γ for $i = 2$ and $1/\gamma$ for $i = 1$. Therefore,

$$\frac{(1-\delta_2)u_2(1-z_1^{M*})}{(1-\delta_1)u_1(z_1^{M*})} = \frac{1}{\gamma} \textit{ and } \frac{(1-\delta_1)u_1(1-z_2^{M*})}{(1-\delta_2)u_2(z_2^{M*})} = \gamma.$$

Now $y \in \xi_\gamma^*(u)$ implies that $y = u(z)$ such that $d(z) = 0$ and

$$\frac{u_2(1-z_1^{M*})}{u_1(z_1^{M*})} \leq \frac{u_2(z_2)}{u_1(z_1)} \leq \frac{u_2(z_2^{M*})}{u_1(1-z_2^{M*})}$$

$$\Leftrightarrow \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma} \leq \frac{u_2(z_2)}{u_1(z_1)} \leq \frac{1-\delta_1}{1-\delta_2}\frac{1}{\gamma}.$$

■

Proof for Corollary 1. Let σ be the Markov perfect equilibrium with the outcome (y, m). Let $z^t = \sigma(h^{t-1})$ for $h^{t-1} \in H$ and $1 \leq t \leq m-1$. By assumption z^t is an incompatible demand profile. In the subgame $g(h^{t-1})$, player i's payoff from following σ is $\delta_i^{m-t} y_i$. Then it must be that $z_i^t \geq 1 - \delta_{-i}^{m-t} y_{-i}$. Otherwise player $-i$ would do better by making the compatible demand $1 - z_i^t$. Set $D = \{z | z_i \geq 1 - \delta_{-i}^{m-t} y_{-i}\}$. So $z^t \in D$.

Next, there cannot exist an incompatible demand $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ such that

$$1 - z_i^t - k_{-i}(z_i^t + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$$

and

$$1 - \hat{z}_{-i} - k_i(z_i^t + \hat{z}_{-i} - 1) > \delta_i^{m-t} y_i.$$

Otherwise, player $-i$ in period t would deviate to the incompatible demand $\hat{z}_{-i}$ and the dominance solvable outcome of the resulting concession game would be (A_i, S_{-i}) with the higher payoff of $\hat{z}_{-i}$. Next, observe that for any $z \in D$ and $\hat{z}_{-i}$ which is incompatible with z_i and greater than $\delta_{-i}^{m-t} y_{-i}$ it follows that $1 - z_i - k_{-i}(z_i + \hat{z}_{-i} - 1) < \delta_{-i}^{m-t} y_{-i}$.

Finally, requiring $1-\hat{z}_{-i}-k_i(z_i^t+\hat{z}_{-i}-1) \leq \delta_i^{m-t} y_i$ to hold for all $\hat{z}_{-i} > \delta_{-i}^{m-t} y_{-i}$ implies that

$$z_i^t \geq \frac{(1-\delta_{-i}^{m-t} y_{-i})(1+k_i) - \delta_i^{m-t} y_i}{k_i}.$$

■

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